14 BALANCING

Introduction

Often an unbalance of forces is produced in rotary or reciprocating machinery due to the inertia forces associated with the moving masses. Balancing is the process of designing or modifying machinery so that the unbalance is reduced to an acceptable level and if possible is eliminated entirely.

A particle or mass moving in a circular path experiences a centripetal acceleration and a force is required to produce it. An equal and opposite force acting radially outwards acts on the axis of rotation and is known as centrifugal force [Fig. 14.1(a)]. This is a disturbing force on the axis of rotation, the magnitude of which is constant but the direction changes with the rotation of the mass.

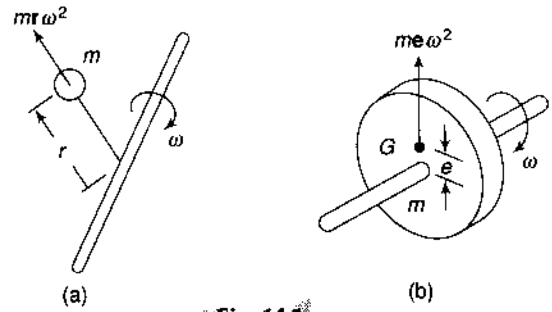


Fig. 14.1

In a revolving rotor, the centrifugal force remains balanced as long as the centre of the mass of the rotor lies on the axis of the shaft. When the centre of mass does not lie on the axis or there is an eccentricity, an unbalanced force is produced (Fig. 14.1b). This type of unbalance is very common. For example, in steam turbine rotors, engine crankshafts, rotary compressors and centrifugal pumps.

Most of the serious problems encountered in high-speed machinery are the direct result of unbalanced forces. These forces exerted on the frame by the moving machine members are time varying, impart vibratory motion to the frame and produce noise. Also, there are human discomfort and detrimental effects on the machine performance and the structural integrity of the machine foundation.

The most common approach to balancing is by redistributing the mass which may be accomplished by addition or removal of mass from various machine members.

There are two basic types of unbalance—rotating unbalance and reciprocating unbalance—which may occur separately or in combination.

14.1 STATIC BALANCING

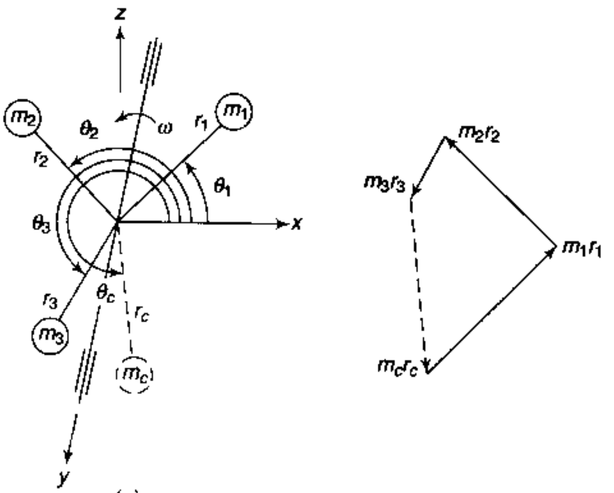
A system of rotating masses is said to be in static balance if the combined mass centre of the system lies on the axis of rotation.

Figure 14.2 shows a rigid rotor revolving with a constant angular velocity of ω rad/s. A number of masses, say three, are depicted by point masses at different radii in the same transverse plane. They may represent different kinds of rotating masses such as turbine blades, eccentric discs, etc. If m_1, m_2 and m_3 are the masses revolving at radii r_1, r_2 and r_3 respectively in the same plane, then each mass produces a centrifugal force acting radially outwards from the axis of rotation. Let **F** be the vector sum of these forces,

$$\mathbf{F} = m_1\mathbf{r}_1\omega^2 + m_2\mathbf{r}_2\omega^2 + m_3\mathbf{r}_3\omega^2$$

The rotor is said to be statically balanced if the vector sum **F** is zero.

If **F** is not zero, i.e., the rotor is unbalanced, then introduce a counterweight (balance weight) of mass m_c, at radius $\mathbf{r}_c$ to balance the rotor so that



(a) (b)

Fig. 14.2

$$m_1\mathbf{r}_1\omega^2 + m_2\mathbf{r}_2\omega^2 + m_3\mathbf{r}_3\omega^2 + m_c\mathbf{r}_c\omega^2 = 0 \tag{14.1}$$

or

$$m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 = 0 \tag{14.1a}$$

The magnitude of either m_c or $\mathbf{r}_c$ may be selected and of the other can be calculated.

In general, if $\Sigma m\mathbf{r}$ is the vector sum of $m_1\mathbf{r}_1$, $m_2\mathbf{r}_2$, $m_3\mathbf{r}_3$, $m_4\mathbf{r}_4$, etc., then

$$\Sigma\, m\mathbf{r} + m_c\mathbf{r}_c = 0 \tag{14.2}$$

The equation can be solved either mathematically or graphically. To solve it mathematically, divide each force into its x and z components,

i.e.,
$$\Sigma\, mr\cos\theta + m_cr_c\cos\theta_c = 0$$
and
$$\Sigma\, mr\sin\theta + m_cr_c\sin\theta_c = 0$$
or
$$m_cr_c\cos\theta_c = -\Sigma mr\cos\theta \tag{i}$$
and
$$m_cr_c\sin\theta_c = -\Sigma mr\sin\theta \tag{ii}$$

Squaring and adding (i) and (ii),

$$m_cr_c = \sqrt{(\Sigma\, mr\cos\theta)^2 + (\Sigma\, mr\sin\theta)^2} \tag{14.3}$$

Dividing (ii) by (i),

$$\tan\theta_c = \frac{-\Sigma\, mr\sin\theta}{-\Sigma\, mr\cos\theta} \tag{14.4}$$

The signs of the numerator and denominator of this function identify the quadrant of the angle.

In graphical solution, vectors, $m_1\mathbf{r}_1$, $m_2\mathbf{r}_2$, $m_3\mathbf{r}_3$, etc., are added. If they close in a loop, the system is balanced. Otherwise, the closing vector will be giving $m_c\mathbf{r}_c$. Its direction identifies the angular position of the countermass relative to the other masses.

Example 14.1 ***Three masses of 8 kg, 12 kg and 15 kg attached at radial distances of 80 mm, 100 mm and 60 mm respectively to a disc on a shaft are in complete balance. Determine the angular positions of the masses of 12 kg and 15 kg relative to the 8-kg mass.***

Solution

$m_1r_1 = 8 \times 80 = 640$

$m_2r_2 = 12 \times 100 = 1200$

$m_3r_3 = 15 \times 60 = 900$

For graphical solution, take a vector representing m_1r_1 of 640-units magnitude along the x-axis. Take the other two vectors through its two ends and complete the triangle. Note that the triangle can be completed in four ways as shown in Fig. 14.3. The results of the four options are

1. $\theta_2 = 227.4°$ and $\theta_3 = 79°$
2. $\theta_2 = 132.6°$ and $\theta_3 = 281°$
3. $\theta_2 = 227.4°$ and $\theta_3 = 79°$
4. $\theta_2 = 132.6°$ and $\theta_3 = 281°$

However, these are only two sets of solutions.

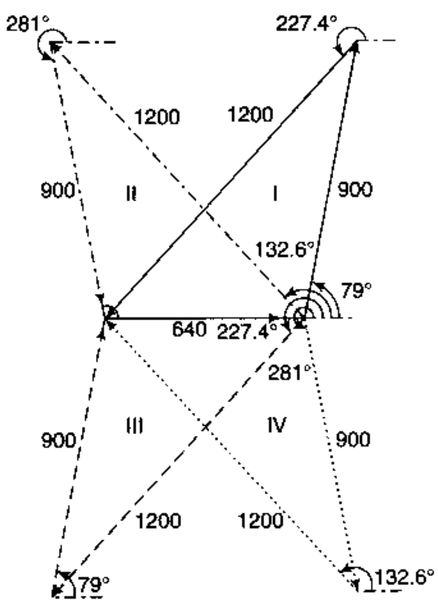


Fig. 14.3

Analytical solution

$\sum mr = 0$

or $\quad 640 \cos 0° + 1200 \cos\theta_2 + 900 \cos\theta_3 = 0$

or $\quad 1200 \cos\theta_2 = -(640 + 900 \cos\theta_3) \qquad$ (i)

and $\quad 640 \sin 0° + 1200 \sin\theta_2 + 900 \sin\theta_3 = 0$

or $\quad 1200 \sin\theta_2 = -900 \sin\theta_3 \qquad$ (ii)

Squaring and adding (i) and (ii),

$$1200^2 = 640^2 + 900^2 \cos^2\theta_3 + 2 \times 640 \times 900 \times \cos\theta_3 + 900^2 \sin^2\theta_3$$

$$= 640^2 + 900^2 + 2 \times 640 \times 900 \times \cos\theta_3$$

$\cos\theta_3 = 0.1913$

or $\quad \theta_3 = 79°$ or $281°$

When $\theta_3 = 79°$, $1200 \sin \theta_2 = -900 \sin 79°$

or $\quad \sin \theta_2 = -0.736$

or $\quad \theta_2 = -47.4°$ or $132.6°$ or $227.4°$

But as $\sin\theta_2$ is negative and $\cos\theta_2$ is also negative which can be found from (i), the corresponding angle $\theta_3 = 227.4°$

In a similar way by taking $\theta_3 = 281°$, θ_2 can be found to be $132.6°$

Example 14.2 ***A circular disc mounted on a shaft carries three attached masses of 4 kg, 3 kg and 2.5 kg at radial distances of 75 mm, 85 mm and 50 mm and at the angular positions of 45°, 135° and 240° respectively The angular positions are measured counter-clockwise from the reference line along the x-axis. Determine the amount of the countermass at a radial distance of 75 mm required for the static balance.***

Solution Figure 14.2 shows the various masses according to the given data.

$m_1r_1 = 4 \times 75 = 300,$

$m_2r_2 = 3 \times 85 = 255,$

$m_3r_3 = 2.5 \times 50 = 125$

$\sum m\mathbf{r} + m_c\mathbf{r}_c = 0$

or $\quad 300 \cos 45° + 255 \cos 135° + 125 \cos 240° + m_cr_c \cos \theta_c = 0$

and $\quad 300 \cos 45° + 255 \sin 135° + 125 \cos 240° + m_cr_c \sin \theta_c = 0$

Squaring, adding and then solving,

$$\therefore \quad m_c r_c = \left[\begin{pmatrix} 300\cos 45° + 255\cos 135° \\ +125\cos 240° \end{pmatrix}^2 + \begin{pmatrix} 300\sin 45° + 255\sin 135° \\ +125\cos 240° \end{pmatrix}^2 \right]^{1/2}$$

or $\quad m_c \times 75 = [(-30.68)^2 + (284.2)^2]^{1/2}$

$\qquad = 285.8$ kg.mm

or $\quad m_c = 3.81$ kg

$$\tan\theta_c = \frac{-284.2}{-(-30.68)} = \frac{-284.2}{+30.68} = -9.26$$

$\therefore \quad \theta_c = 276°12'$

θ_c lies in the fourth quadrant ($\because$ numerator is negative and denominator is positive).

The graphical solution has been carried out in Fig. 14.3(b).

14.2 DYNAMIC BALANCING

When several masses rotate in different planes, the centrifugal forces, in addition to being out of balance, also form couples. A system of rotating masses is in dynamic balance when there does not exist any resultant centrifugal force as well as resultant couple.

In the work that follows, the products of mr and mrl (instead of $mr\omega^2$ and $mrl\omega^2$), usually, have been referred as force and couple respectively as it is more convenient to draw force and couple polygons with these quantities.

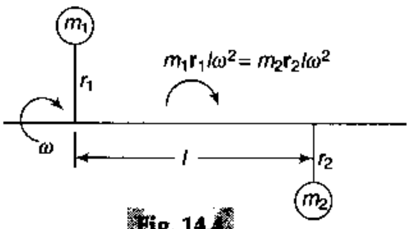


Fig. 14.4

If m_1 and m_2 are two masses (Fig. 14.4) revolving diametrically opposite to each other in different planes such that $m_1r_1 = m_2r_2$, the centrifugal forces are balanced, but an unbalanced couple of magnitude m_1r_1l $(= m_2r_2l)$ is introduced. The couple acts in a plane that contains the axis of rotation and the two masses. Thus, the couple is of constant magnitude but variable direction.

14.3 TRANSFERENCE OF A FORCE FROM ONE PLANE TO ANOTHER

Let m be the mass at radius r rotating in a plane at a distance l from another plane (Fig. 14.5). The equilibrium of the system does not change if two equal and opposite forces $F_1 = F_2$ $(= mr)$ are added in the latter plane. The net effect would be a single force F_1 $(= mr)$ in the second plane having the direction of the original force along with a couple mrl formed by the forces mr and F_2 in a plane containing these forces and the shaft. As the moment of a couple is the same about any point in its plane (equal to the product of one of the forces and the arm), the couple may be assumed to rotate the shaft about the point O.

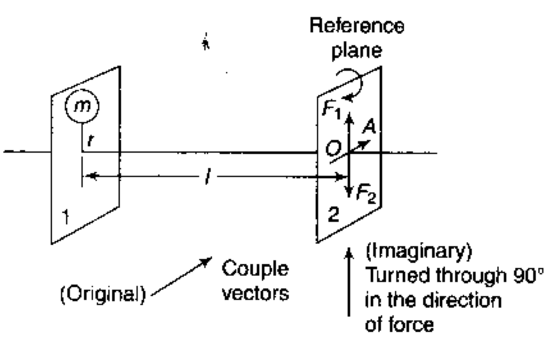


Fig. 14.5

The axis of rotation of the couple is thus a line OA drawn perpendicular to the shaft through O. A line drawn parallel to the axis and to a suitable scale can represent the couple vectorially, the sense of rotation of which is given by the right-hand corkscrew rule, i.e., for a clockwise couple, the direction is to be away from the viewer. However, in balancing problems, it becomes convenient if the couple vectors are drawn by turning them through 90°, i.e., by drawing them parallel to the force vectors. This does not affect their relative positions.

A plane passing through a point such as O and perpendicular to the axis of the shaft is called a *reference plane*. Other masses acting in different planes can be transferred to the reference plane in a similar manner as discussed above.

14.4 BALANCING OF SEVERAL MASSES IN DIFFERENT PLANES

Let there be a rotor revolving with a uniform angular velocity ω [Fig. 14.6(a)]. m_1, m_2 and m_3 are the masses attached to the rotor at radii r_1, r_2 and r_3 respectively. The masses m_1, m_2 and m_3 rotate in planes 1, 2 and 3 respectively. Choose a reference plane at O so that the distances of the planes 1, 2 and 3 from O are l_1, l_2 and l_3 respectively.

Transference of each unbalanced force to the reference plane introduces the like number of forces and couples.

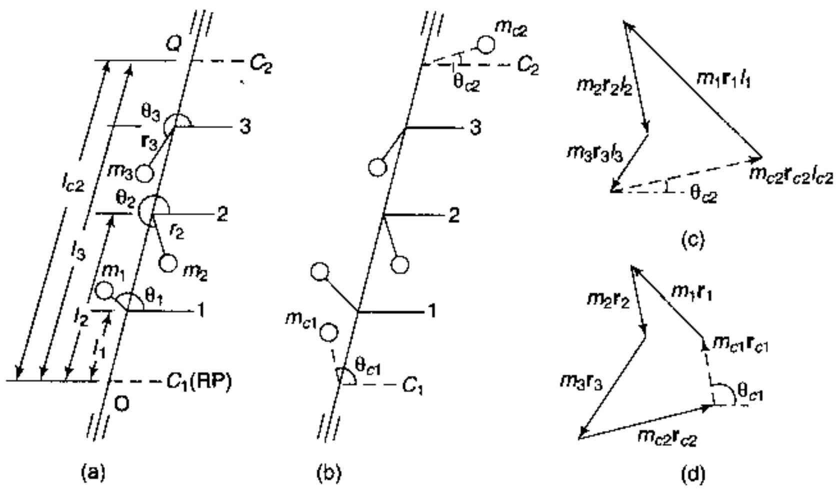


Fig. 14.6

The unbalanced forces in the reference plane are $m_1\mathbf{r}_1\omega^2$, $m_2\mathbf{r}_2\omega^2$ and $m_3\mathbf{r}_3\omega^2$ acting radially outwards.

The unbalanced couples in the reference plane are $m_1\mathbf{r}_1\omega^2 l_1$, $m_2\mathbf{r}_2\omega^2 l_2$ and $m_3\mathbf{r}_3\omega^2 l_3$ which may be represented by vectors parallel to the respective force vectors, i.e., parallel to the respective radii of m_1, m_2 and m_3.

For complete balancing of the rotor, the resultant force and the resultant couple both should be zero, i.e.,

$$m_1\mathbf{r}_1\omega^2 + m_2\mathbf{r}_2\omega^2 + m_3\mathbf{r}_3\omega^2 = 0 \tag{14.5}$$

and

$$m_1\mathbf{r}_1 l_1\omega^2 + m_2\mathbf{r}_2 l_2\omega^2 + m_3\mathbf{r}_3 l_3\omega^2 = 0 \tag{14.6}$$

If the Eqs (14.5) and (14.6) are not satisfied, then there are unbalanced forces and couples. A mass placed in the reference plane may satisfy the force equation but the couple equation is satisfied only by two equal forces in different transverse planes. Thus, in general, two planes are needed to balance a system of rotating masses.

Therefore, in order to satisfy Eqs (14.5) and (14.6), introduce two counter-masses m_{c1} and m_{c2} at radii r_{c1} and r_{c2} respectively. Then Eq. (14.5) may be written as

$$m_1\mathbf{r}_1\omega^2 + m_2\mathbf{r}_2\omega^2 + m_3\mathbf{r}_3\omega^2 + m_{c1}\mathbf{r}_{c1}\omega^2 + m_{c2}\mathbf{r}_{c2}\omega^2 = 0 \tag{14.7}$$

or

$$m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 + m_{c1}\mathbf{r}_{c1} + m_{c2}\mathbf{r}_{c2} = 0 \tag{14.7a}$$

In general,

$$\Sigma m\mathbf{r} + m_{c1}\mathbf{r}_{c1} + m_{c2}\mathbf{r}_{c2} = 0 \tag{14.8}$$

Let the two countermasses be placed in transverse planes at axial locations O and Q, i.e., the countermass m_{c1} be placed in the reference plane and the distance of the plane of m_{c2} be l_{c2} from the reference plane.

Equation (14.6) modifies to (taking moments about O)

$$m_1\mathbf{r}_1 l_1\omega^2 + m_2\mathbf{r}_2 l_2\omega^2 + m_3\mathbf{r}_3 l_3\omega^2 + m_{c2}\mathbf{r}_{c2} l_{c2}\omega^2 = 0 \tag{14.9}$$

or

$$m_1\mathbf{r}_1 l_1 + m_2\mathbf{r}_2 l_2 + m_3\mathbf{r}_3 l_3 + m_{c2}\mathbf{r}_{c2} l_{c2} = 0 \tag{14.9a}$$

In general,

$$\Sigma m\mathbf{r}l + m_{c2}\mathbf{r}_{c2} l_{c2} = 0 \tag{14.10}$$

Thus, Eqs (14.8) and (14.10) are the necessary conditions for dynamic balancing of the rotor. Again the equations can be solved mathematically or graphically.

Dividing Eq. (14.10) into component form

$$\Sigma m\mathbf{r}l\cos\theta + m_{c2}\mathbf{r}_{c2}l_{c2}\cos\theta_{c2} = 0$$

and

$$\Sigma m\mathbf{r}l\sin\theta + m_{c2}\mathbf{r}_{c2}l_{c2}\sin\theta_{c2} = 0$$

or

$$m_{c2}\mathbf{r}_{c2}l_{c2}\cos\theta_{c2} = -\Sigma m\mathbf{r}l\cos\theta \tag{i}$$

and

$$m_{c2}\mathbf{r}_{c2}l_{c2}\sin\theta_{c2} = -\Sigma m\mathbf{r}l\sin\theta \tag{ii}$$

Squaring and adding (i) and (ii)

$$m_{c2}r_{c2}l_{c2} = \sqrt{(\Sigma\, mrl\cos\theta)^2 + (\Sigma mrl\sin\theta)^2} \tag{14.11}$$

Dividing (ii) by (i),

$$\tan\theta_{c2} = \frac{-\Sigma\, mrl\sin\theta}{-\Sigma\, mrl\cos\theta} \tag{14.12}$$

After obtaining the values of m_{c2} and θ_{c2} from the above equations, solve Eq. (14.8) by taking its components,

$$m_{c1}r_{c1} = \sqrt{(\Sigma\, mr\cos\theta + m_{c2}r_{c2}\cos\theta_{c2})^2 + (\Sigma\, mrl\sin\theta + m_{c2}r_{c2}\sin\theta_{c2})^2} \tag{14.13}$$

and

$$\tan\theta_{c1} = \frac{-(\Sigma\, mr\sin\theta + m_{c2}r_{c2}\sin\theta_{c2})}{-(\Sigma\, mr\cos\theta + m_{c2}r_{c2}\cos\theta_{c2})} \tag{14.14}$$

To solve Eqs (14.8) and (14.10) graphically, Eq. (14.10) is solved first and a couple polygon is made by adding the known vectors and considering each vector parallel to the radial line of the mass. Then the closing vector will be $m_{c2}\mathbf{r}_{c2}l_{c2}$, the direction of which specifies the angular position of the countermass m_{c2} [Fig. 14.6(c)] in the plane at the point Q. Then solve Eq. (14.8) and make a force polygon by adding the known vectors (along with the vector $m_{c2}\mathbf{r}_{c2}$). The closing vector is $m_{c1}\mathbf{r}_{c1}$, identifying the magnitude and the direction of the countermass m_{c1} [Fig. 14.6(d)]. Figure 14.6(b) represents the position of the balancing masses on the rotating shaft.

Example 14.3 *A rotating shaft carries three unbalanced masses of 4 kg, 3 kg and 2.5 kg at radial distances of 75 mm, 85 mm and 50 mm and at the angular positions of 45°, 135° and 240° respectively. The second and the third masses are in the planes at 200 mm and 375 mm from the plane of the first mass. The angular positions are measured counter-clockwise from the reference line along x-axis and viewing the shaft from the first mass end.*

The shaft length is 800 mm between bearings and the distance between the plane of the first mass and the bearing at that end is 225 mm. Determine the amount of the countermasses in planes at 75 mm from the bearings for the complete balance of the shaft. The first countermass is to be in a plane between the first mass and the bearing and the second mass in a plane between the third mass and the bearing at that end.

Solution Figure 14.7(a) shows the planes of unbalanced masses as well as the planes of the countermasses. Plane C_1 is to be taken as the reference plane and the various distances are to be considered from this plane.

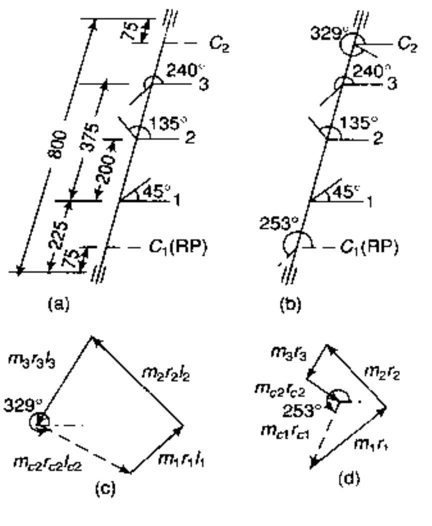


Fig. 14.7

Analytical solution

$l_{c2} = (800 - 75 \times 2) = 650$ mm

$l_1 = 225 - 75 = 150$ mm

$l_2 = 150 + 200 = 350$ mm

$l_3 = 150 + 375 = 525$ mm

$m_1 r_1 l_1 = 4 \times 75 \times 150 = 45\,000 \quad m_1 r_1 = 4 \times 75 = 300$

$m_2 r_2 l_2 = 3 \times 85 \times 350 = 89\,250 \quad m_2 r_2 = 3 \times 85 = 255$

$m_3 r_3 l_3 = 2.5 \times 50 \times 525 = 65\,625 \quad m_3 r_3 = 2.5 \times 50 = 125$

$\Sigma m\mathbf{r}l + m_{c2}\mathbf{r}_{c2}\, l_{c2} = 0$

or $4500 \cos 45° + 89\,250 \cos 135° + 65\,625 \cos 240° + m_{c2}\mathbf{r}_{c2} l_{c2} \cos\theta_{c2} = 0$

and $45000 \sin 45° + 89\,250 \sin 135° + 65\,625 \sin 240° + m_{c2}\mathbf{r}_{c2} l_{c2} \sin\theta_{c2} = 0$

Squaring, adding and then solving,

$$m_{c2} r_{c2} l_{c2} = \left[\begin{pmatrix} 45\,000 \cos 45° + 89\,250 \\ \cos 135° + 65\,625 \cos 240° \end{pmatrix}^2 + \begin{pmatrix} 45\,000 \sin 45° + 89\,250 \\ \sin 135° + 65\,625 \sin 240° \end{pmatrix}^2 \right]^{1/2}$$

$$= [(-64\,102)^2 + (38\,096)^2]^{1/2}$$

or $\quad m_{c2} \times 40 \times 650 = 74\,568$

$m_{c2} = \underline{2.868 \text{ kg}}$

$$\tan \theta_{c2} = \frac{-38\,096}{-(-64\,102)} = -0.594$$

$\theta_{c2} = 329.3°$ or $\underline{329°18'}$

Now,

$\Sigma m\mathbf{r} + m_{c1}\mathbf{r}_{c1} + m_{c2}\mathbf{r}_{c2} = 0$

or $\quad 300 \cos 45° + 255 \cos 135° + 125 \cos 240° + m_{c1}\mathbf{r}_{c1} \cos \theta_1 + 2.868 \times 40 \cos 329.3 = 0$

and $\quad 300 \sin 45° + 255 \sin 135° + 125 \sin 240° + m_{c1}\mathbf{r}_{c1} \sin \theta_1 + 2.868 \times 40 \sin 329.3 = 0$

Squaring, adding and then solving,

$$m_{c1} r_{c1} = \left[\begin{array}{l} (300 \cos 45° + 255 \cos 135° \\ + 125 \cos 240° + 2.868 \\ \times 40 \cos 329.3°)^2 + \\ (300 \sin 45° + 255 \sin 135° \\ + 125 \sin 240° + 2.868 \\ \times 40 \sin 329.3°)^2 \end{array} \right]^{1/2}$$

$m_{c1} \times 75 = [(67.96)^2 + (225.62)^2]^{1/2} = 235.63$

$m_{c1} = \underline{3.14 \text{ kg}}$

$$\tan \theta_{c1} = \frac{-225.62}{-67.96} = 3.32; \theta_{c1} = 253.2° \text{ or } \underline{253°12'}$$

Graphical solution

The graphical solution has also been shown in Figs 14.7(c) and (d). From Fig. 14.7(c),

$m_{c2}r_{c2}l_{c2} = 74\,000$

$\therefore \quad m_{c2} = \dfrac{74\,000}{40 \times 650} = 2.846 \text{ kg at } \underline{329°}$

From Fig. 14.7(d),

$m_{c1}r_{c1} = 235,$

$\therefore m_{c1} = \dfrac{235}{75} = 3.13 \text{ kg at } \underline{253°}$

Figure 14.7(b) represents the position of the balancing masses on the rotating shaft.

Solution by using complex numbers

$m_1r_1l_1 \angle\theta_1 = (4 \times 75 \times 150) \angle 45° = 45\,000 \angle 45°$
$= 31\,820 + j\,31\,820$

$m_2r_2l_2 \angle\theta_2 = (3 \times 85 \times 350) \angle 135° = 89\,250 \angle 135°$
$= -63\,109 + j\,63\,109$

$m_3r_3l_3 \angle\theta_3 = (2.5 \times 50 \times 525) \angle 240° = -65\,625 \angle 240°$
$= -32\,813 - j\,56\,833$

Now,

$m_1r_1l_1 \angle\theta_1 + m_2r_2l_2 \angle\theta_2 + m_3r_3l_3 \angle\theta_3 + m_{c2}r_{c2}l_{c2} \angle\theta_{c2} = 0$

$(31\,820 + j\,31\,820) + (-63\,109 + j\,63\,109) + (-32\,813 - j\,56\,833) + m_{c2}r_{c2}l_{c2} \angle\theta_{c2} = 0$

$m_{c2}r_{c2}l_{c2} \angle\theta_{c2} = 64\,102 - j\,38\,096 = 74\,568 \angle 329.3°$

$m_{c2} \times 40 \times 650 = 74\,568$

$mc_2 = \underline{2.868}$ kg

Similarly,

$m_1r_1\angle\theta_1 = (4 \times 75) \angle 45° = 300 \angle 45° = 212.1 + j212.1$

$m_2r_2\angle\theta_2 = (3 \times 85) \angle 135° = 255 \angle 135° = -180.3 + j180.3$

$m_3r_3\angle\theta_3 = (2.5 \times 50) \angle 240° = 125 \angle 240° = -62.5 - j108.3$

$m_{c2}r_{c2} \angle\angle\theta_{c2} = (2.868 \times 40) \angle 329.3° = 114.72 \angle 329.3° = 98.6 - j\,58.6$

Now,

$m_1r_1\angle\theta_1 + m_2r_2\angle\theta_2 + m_3r_3\angle\theta_3 + m_{c2}r_{c2} \angle\theta_{c2} + m_{c1}r_{c1} \angle\theta_{c1} = 0$

$(212.1 + j\,212.1) + (-180.3 + j\,180.3) + (-62.5 - j\,108.3) + (98.6 - j\,58.6) + m_{c2}r_{c2} \angle 329.3° = 0$

$m_{c1}r_{c1} \angle\theta_{c1} = -67.9 - j\,225.5$
$= 235.5 \angle 253.2°$

or

$m_{c1} \times 75 = 235.63$

$m_{c1} = \underline{3.14 \text{ kg}}$

Example 14.4 ***A shaft supported in bearings that are 1.6 m apart projects 400 mm beyond bearings at each end. It carries three pulleys one at each end and one at the centre of its length. The masses of the end pulleys are 40 kg and 22 kg and their centres of mass are at 12 mm and 18 mm respectively from the shaft axes. The mass of the centre pulley is 38 kg and its centre of mass is 15 mm from the shaft axis. The pulleys are arranged in a manner that they give static balance. Determine the***

(i) relative angular positions of the pulleys

(ii) dynamic forces developed on the bearings when the shaft rotates at 210 rpm

Solution

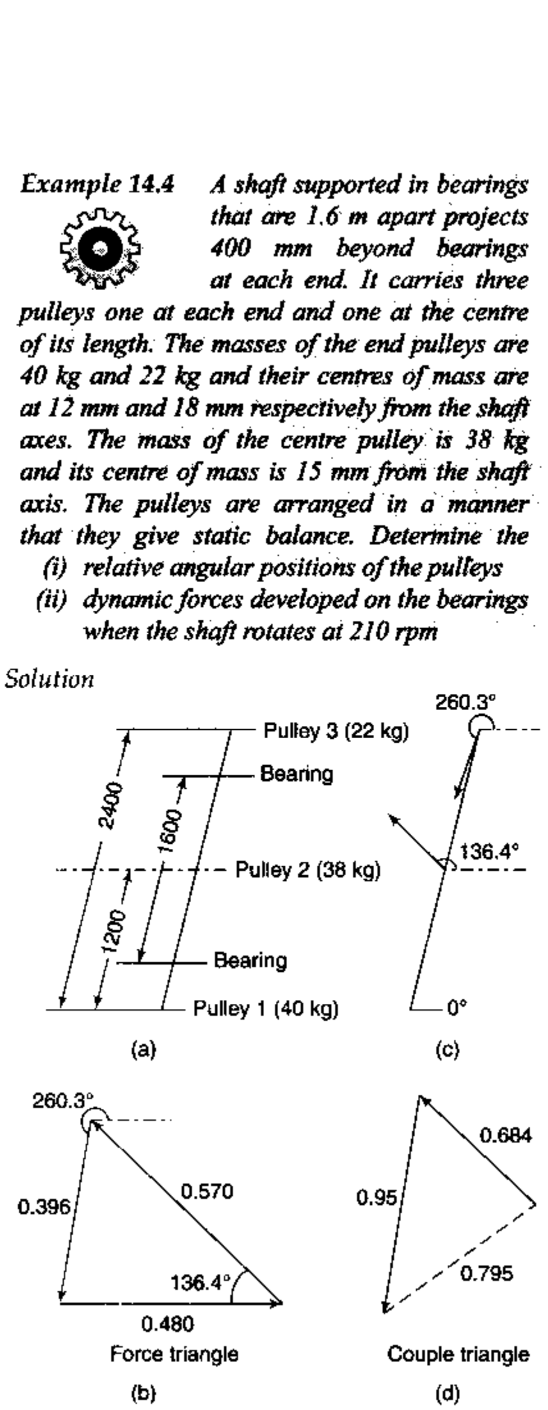


Fig. 14.8

Figure 14.8(a) shows the planes of the three pulleys as well as of the two bearings.

Let the plane of the pulley 1 be the reference plane.

$$m_1r_1 = 40 \times 0.012 = 0.48$$
$$m_2r_2l_2 = 38 \times 0.015 \times 1.2 = 0.684$$
$$m_2r_2 = 38 \times 0.015 = 0.57$$
$$m_3r_3l_3 = 22 \times 0.018 \times 2.4 = 0.95$$
$$m_3r_3 = 22 \times 0.018 = 0.396$$

Complete the force triangle as the three sides are known [Fig. 14.8(b)]. The mass at the plane 1 is chosen at 0° angle. By completing it, the directions of the other two masses are known which have been marked in Fig. 14.8(c).

Now, as the shaft is in complete static balance, there is only unbalanced couple which is to be the same about all planes. Thus, reactions due to the unbalanced couple are to be equal and opposite on the two bearings.

To find the magnitude of the unbalanced couple, add the two couple vectors as shown in Fig. 14.8(d). The closing side shown in dotted line represents the magnitude of the unbalanced couple.

The magnitude, $mrl = 0.795$ on measurement.

$\therefore$ unbalanced couple = $mr\omega^2.l$

$$= 0.795 \times \left(\frac{2\pi \times 210}{60}\right)^2 = 384.5 \text{ N.m}$$

The reaction on each bearing $= \dfrac{384.5}{1.6} = 240.3$ N

Example 14.5 ***Four masses A, B, C and D carried by a rotating shaft at radii 80 mm, 100 mm, 160 mm and 120 mm respectively are completely balanced. Masses B, C and D are 8 kg, 4 kg and 3 kg respectively. Determine the mass A and the relative angular positions of the four masses if the planes are spaced 500 mm apart.***

Solution Figure 14.9(a) shows the planes of the four masses. Let plane A be the reference plane.

$$m_ar_a = m_1 \times 0.08 = 0.08\, m_1$$
$$m_br_bl_b = 8 \times 0.1 \times 0.5 = 0.4$$
$$m_br_b = 8 \times 0.1 = 0.8$$
$$m_cr_cl_c = 4 \times 0.16 \times 1 = 0.64$$
$$m_cr_c = 4 \times 0.16 = 0.64$$
$$m_dr_dl_d = 3 \times 0.12 \times 1.5 = 0.54$$
$$m_dr_d = 3 \times 0.12 = 0.36$$

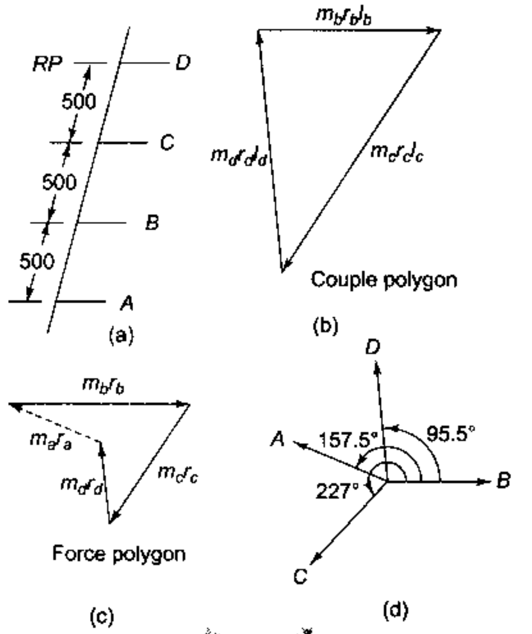


Fig. 14.9

On taking the plane A as the reference plane, there are only three couple vectors. Assuming the direction of any of the masses B, C or D at 0° angle, a vector triangle can be made as shown in Fig. 14.9(b). As the shaft is in complete balance, the arrowheads may be put in the same order. This provides the directions of masses of C and D relative to that of B.

Now, as the shaft is in complete static balance also, a force polygon may be completed as shown in Fig. 14.9(c). The closing side provides the magnitude of the mass radius.

The magnitude, $m_ar_a = 0.444$ on measurement.

or $m_a = 0.444/0.08 = 5.55$ kg

The angular positions of masses A, C and D relative to that of the mass B are 95.5°, 157.5° and 227° counter-clockwise as shown in Fig. 14.9(d).

Example 14.6 ***A rotor is completely balanced when masses of 2 kg and 1.2 kg are added temporarily in planes A and D each at 200 mm radius as shown in Fig. 14.10(a). The balanced mass in the plane A is along the x-axis whereas in the plane D, it is at 120° counter-clockwise.***

It is desired that the actual balancing is to be done by adding permanent masses in planes B and C, each at 120 mm radius. Determine the magnitudes and the directions of the masses B and C.

Solution

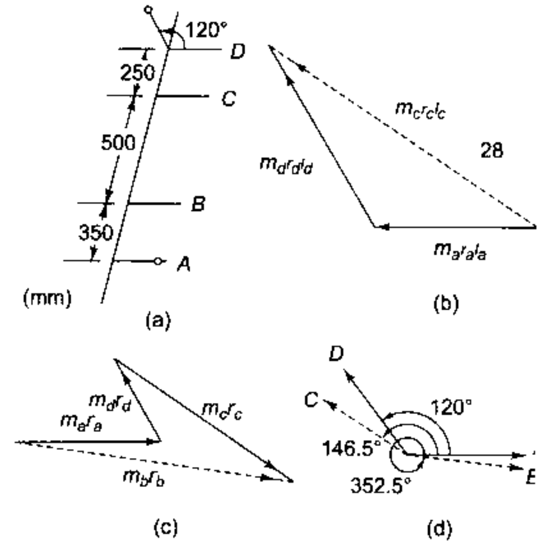


Fig. 14.10

It is given that the rotor is completely balanced with the temporary masses in planes A and D. It is required to find the masses and their directions in planes B and C which provide the same resultant force and the couple so that the rotor is again balanced.

Unbalanced couple about a plane at B

$$= m_a \mathbf{r}_a(-l_a) + m_d \mathbf{r}_d l_d = -m_a \mathbf{r}_a l_a + m_d \mathbf{r}_d l_d$$

where $m_a r_a l_a = 2 \times 0.2 \times (-0.35) = -0.14$

and $m_d r_d l_d = 1.2 \times 0.2 \times 0.75 = 0.18$

Assuming the masses required in planes B and C be m_b and m_c respectively, then the magnitude of the couples due to these masses must be equal to the above couple, i.e.,

$$0 + m_c \mathbf{r}_c l_c = -m_a \mathbf{r}_a l_a + m_d \mathbf{r}_d l_d$$

Thus, the resultant of the two vectors on the right-hand side provides the vector $m_c \mathbf{r}_c l_c$ as shown in Fig. in 14.10(b). On measurement, $m_c \mathbf{r}_c l_c = 0.28$ at 146.5°

$$\therefore m_c = \frac{0.28}{0.5 \times 0.12} = 4.67 \text{ kg at } 146.5°$$

Similarly, the vector sum of the forces due to masses at B and C must be equal to the vector sum of the forces due to masses at A and D, i.e.,

$$m_b \mathbf{r}_b + m_c \mathbf{r}_c = m_a \mathbf{r}_a + m_d \mathbf{r}_d$$

or $\quad m_b \mathbf{r}_b = m_a \mathbf{r}_a + m_d \mathbf{r}_d - m_c \mathbf{r}_c$

Now, $\quad m_a r_a = 2 \times 0.2 = 0.4$

$m_d r_d = 1.2 \times 0.2 = 0.24$

$m_c r_c = 4.67 \times 0.12 = 0.56$

Thus the resultant of the three vectors on the right-hand side provides the vector $m_b \mathbf{r}_b$ as shown in Fig. 14.10(c). On measurement, $m_b \mathbf{r}_b = 0.75$ at 352.5°

$$\therefore m_b = \frac{0.75}{0.12} = 6.25 \text{ kg at } 352.5°$$

Figure 14.10(d) shows angular positions of all the four masses.

Example 14.7 *Four masses A, B, C and D are completely balanced. Masses C and D make angles of 90° and 195° respectively with that of mass B in the counter-clockwise direction. The rotating masses have the following properties:*

$m_b = 25$ kg	$r_a = 150$ mm
$m_c = 40$ kg	$r_b = 200$ mm
$m_d = 35$ kg	$r_c = 100$ mm
	$r_d = 180$ mm

Planes B and C are 250 mm apart. Determine the

(i) mass A and its angular position with that of mass B

(ii) positions of all the planes relative to plane of mass A

Solution Refer Fig. 14.11(a).

$m_b r_b = 25 \times 100 = 5000$

$m_c r_c = 40 \times 100 = 4000$

$m_d r_d = 35 \times 180 = 6300$

For complete balance, taking $\theta_b = 0°$

$\sum mr \cos\theta = 0$ and $\sum mr \sin\theta = 0$

i.e., $m_a \times 150 \times \cos\theta_a + 5000 \cos 0° + 4000 \cos 90° + 6300 \cos 195° = 0$

or $m_a \times 150 \times \cos\theta_a + 5000 + 0 - 6085 = 0$

or $150\, m_a \cos\theta_a = 1085$ (i)

and $m_a \times 150 \times \sin\theta_a + 5000 \sin 0° + 4000 \sin 90° + 6300 \sin 195° = 0$

or $m_a \times 150 \times \sin\theta_a + 0 + 4000 - 1631 = 0$

or $150\, m_a \sin\theta_a = -2369$ (ii)

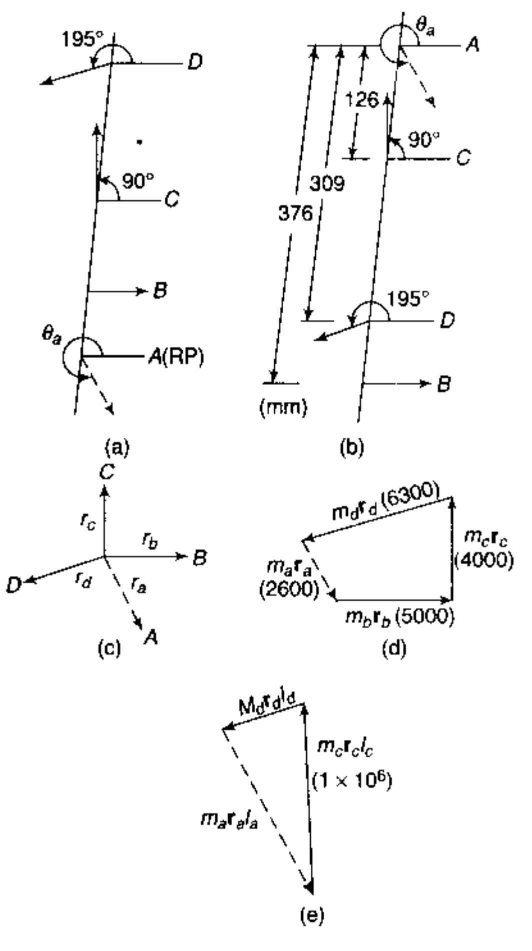


Fig. 14.11

Squaring and adding (i) and (ii),

$22\,500\, m_a^2 = (1085)^2 + (-2369)^2$

or $m_a^2 = 30\,175$

or $m_a = \underline{17.37 \text{ kg}}$

Dividing (ii) by (i),

$$\tan\theta_a = \frac{-236.9}{108.5} = -2.184$$

or $\theta_a = 294.6°$ or $\underline{294°36'}$

For complete balance, the couple equations are

$\sum mrl \cos\theta = 0$ and $\sum mrl \sin\theta = 0$

Taking A as the reference plane,

$5000\, l_b \cos 0° + 4000\, l_c \cos 90° + 6300\, l_d \cos 195° = 0$

or $5000\, l_b = 6085\, l_d$

or $l_b = 1.217\, l_d$

and $5000\, l_b \sin 0° + 4000\, l_c \sin 90° + 6300\, l_d \sin 195° = 0$

or $4000\, l_c = 1631\, l_d$

or $l_c = 0.4078\, l_d$

or $l_b + 250 = 0.4078\, l_d$

or $1.217\, l_d + 250 = 0.4078\, l_d$

or $0.8092\, l_d = -250$

or $l_d = -309$ mm

$l_b = 1.217\, l_d = 1.217 \times (-309) = -376$ mm

$l_c = l_b + 250 = -376 + 250 = -126$ mm

The correct positions of the planes have been shown in Figs. 14.11 (b) and (c).

To solve the problem graphically, $m_a \mathbf{r}_a$ is obtained from the vector sum of $m_b \mathbf{r}_b$, $m_c \mathbf{r}_c$ and $m_d \mathbf{r}_d$ [Fig. 14.11(d)]. On measuring,

$m_a \mathbf{r}_a = 2600$,

$$\therefore m_a = \frac{2600}{150} = \underline{17.3 \text{ kg}} \text{ and } \theta_a = \underline{294.5°}$$

Now, $m_a \mathbf{r}_a l_a = 4000 \times 250 = 1 \times 10^6$, taking B as the reference plane. Take the vector $m_c \mathbf{r}_c l_c$ and from its two ends, draw lines parallel to $m_a \mathbf{r}_a$ and $m_d \mathbf{r}_d$. Thus, forming a triangle [Fig. 14.11(e)]. Measuring the two sides,

$$m_a \mathbf{r}_a l_a = 985\,000,\ l_a = \frac{985\,000}{17.3 \times 150} = \underline{379 \text{ mm}}$$

$$m_d \mathbf{r}_d l_d = 437\,000,\ l_d = \frac{437\,000}{6300} = \underline{69 \text{ mm}}$$

l_a and l_d establish the relative positions of the planes.

14.5 FORCE BALANCING OF LINKAGES

Balancing of a linkage implies that the total centre of its mass remains stationary so that the vector sum of all the frame forces always remains zero. Figure 14.12 shows a four-link mechanism. a, b, c and d represent the magnitudes of the links AB, BC, CD and DA respectively. The link masses are m_a, m_b and m_c, located at G_a, G_b and G_c respectively. Let the coordinates g_i, ϕ_i describe the position of these points within each link.

As in any configuration of the mechanism, the links of the mechanism can be considered as vectors. Thus,

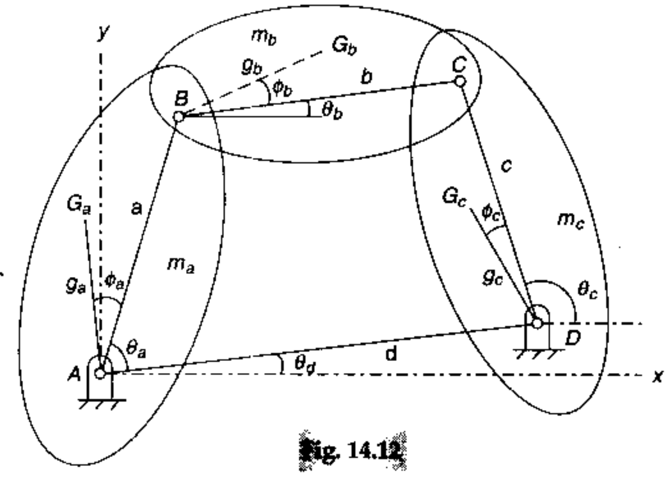


Fig. 14.12

$$ae^{i\theta_a} + be^{i\theta_b} - ce^{i\theta_c} - de^{i\theta_d} = 0 \quad (14.15)$$

or

$$e^{i\theta_b} = \frac{1}{b}(de^{i\theta_d} - ae^{i\theta_a} + ce^{i\theta_c}) \quad (14.15a)$$

Let G be the centre of mass for the system of moving links and let **g** define the position of G with respect to origin A.

Total mass of moving links, $m = m_a + m_b + m_c$ (14.16)

Then for the centre of mass of the entire system to remain stationary at a point, the following expression must be a constant (acceleration due to weight is constant).

$$m\mathbf{g} = m_a\mathbf{g}_a + m_b\mathbf{g}_b + m_c\mathbf{g}_c \quad (14.17)$$

where $\mathbf{g}_a$, $\mathbf{g}_b$ and $\mathbf{g}_c$ are the vectors representing the positions of masses m_a, m_b and m_c respectively, w.r.t. A.

$$m\mathbf{g} = m_a g_a e^{i(\theta_a+\varphi_a)} + m_b[ae^{i\theta_a} + g_b e^{i(\theta_b+\varphi_b)}] + m_c[de^{i\theta_d} + g_c e^{i(\theta_c+\varphi_c)}]$$

$$= m_a g_a e^{i\theta_a} e^{i\varphi_a} + m_b a e^{i\theta_a} + m_b g_b e^{i\theta_b} e^{i\varphi_b} + m_c de^{i\theta_d} + m_c g_c e^{i\theta_c} e^{i\varphi_c}$$

Inserting the value of $e^{i\theta_b}$ from (14.15a)

$$mg = m_a g_a e^{i\theta_a} e^{i\varphi_a} + m_b a e^{i\theta_a} + m_b g_b \frac{1}{b}(de^{i\theta_d} - ae^{i\theta_a} + ce^{i\theta_c})e^{i\varphi_b} + m_c de^{i\theta_d} + m_c g_c e^{i\theta_c} e^{i\varphi_c}$$

$$= \left(m_a g_a e^{i\varphi_a} + m_b a - m_b g_b \frac{a}{b} e^{i\varphi_b}\right)e^{i\theta_a} + \left(m_c g_c e^{i\varphi_c} + m_b g_b \frac{c}{b} e^{i\varphi_b}\right)e^{i\theta_c} + \left(m_c d + m_b g_b \frac{d}{b} e^{i\varphi_b}\right)e^{i\theta_d}$$

The centre of mass can be made stationary at the position $\mathbf{g} = \left(m_c d + m_b g_b \frac{d}{b} e^{i\varphi_b}\right)e^{i\theta_d}$

if the remaining two terms in the brackets can be made zero. Let the vector $\mathbf{g}_a'$ represent the position of the countermass m_a' to be added to the input link and vector $\mathbf{g}_c'$ represent the position of the countermass m_c' to be added to the output link to have complete force balancing.

Thus the equations may be written as

$$m_a g_a e^{i\varphi_a} + m_b a - m_b g_b \frac{a}{b} e^{i\varphi_b} + m_a' g_a' e^{i\varphi_a'} = 0 \tag{14.18}$$

and
$$m_c g_c e^{i\varphi_c} + m_b g_b \frac{c}{b} e^{i\varphi_b} + m_c' g_c' e^{i\varphi_c'} = 0 \tag{14.19}$$

from which magnitudes and the locations of the countermasses can be obtained.

Example 14.8 ***The following data relate to a four-link mechanism:***

$a = 55$ mm	$m_a = 0.045$ kg
$b = 165$ mm	$m_b = 0.13$ kg
$c = 80$ mm	$m_c = 0.05$ kg
$d = 150$ mm	
$g_a = 28$ mm	$\phi_a = 0°$
$g_b = 85$ mm	$\phi_b = 15°$
$g_c = 42$ mm	$\phi_c = 0°$

Complete force balancing by adding countermasses to the input and the output links is desired. Determine the mass-distance values and angular position of each counter mass.

Solution We have

$$m_a g_a e^{i\varphi_a} + m_b a - m_b g_b \frac{a}{b} e^{i\varphi_b} + m_a' g_a' e^{i\varphi_a'} = 0$$

$0.045 \times 0.028 \cos 0° + 0.13 \times 0.055 - 0.13 \times 0.085\ (0.055/0.165) \cos 15° + m_a' g_a' \cos \phi_a' = 0$

$0.001\,26 + 0.007\,15 - 0.003\,56 + m_a' g_a' \cos \phi_a' = 0$

$m_a' g_a' \cos \phi_a' = -\,0.004\,853$ (i)

$0.045 \times 0.028 \sin 0° - 0.13 \times 0.085\ (0.055/0.165) \sin 15° + m_a' g_a' \sin \phi_a' = 0$

$0 - 0.000\,954 + m_a' g_a' \sin \phi_a' = 0$

$m_a' g_a' \sin \phi_a' = 0.000\,954$ (ii)

Squaring and adding (i) and (ii),

$(m_a' g_a')^2 = 0.000\,024\,46$

or $m_a' g_a' = \underline{0.004\,946 \text{ kg.m}}$

Dividing (ii) by (i),

$$\tan \phi_a' = \frac{0.000954}{-0.004853} = 0.1966$$

$\phi_a' = \underline{168.9°}$

$$m_c g_c e^{i\varphi_c} + m_b g_b \frac{c}{b} e^{i\varphi_b} + m_c' g_c' e^{i\varphi_c'} = 0$$

$0.05 \times 0.042 \cos 0° + 0.13 \times 0.085\ (0.08/0.165) \cos 15° + m_c' g_c' \cos \phi_c' = 0$

$0.0021 + 0.005\,175 + m_c' g_c' \cos \phi_c' = 0$

$m_c' g_c' \cos \phi_c' = -\,0.007\,275$ (iii)

$0.05 \times 0.042 \sin 0° + 0.13 \times 0.085\ (0.08/0.165) \sin 15° + m_c' g_c' \sin \phi_c' = 0$

$0 + 0.001\,387 + m_c' g_c' \sin \phi_c' = 0$

$m_c' g_c' \sin \phi_c' = -\,0.001\,387$ (iv)

Squaring and adding (iii) and (iv),

$(m_c' g_c')^2 = 0.000\,063\,86$

or $m_c' g_c' = \underline{0.007\,41 \text{ kg.m}}$

Dividing (iv) by (iii),

$$\tan \phi_c' = \frac{-.001\,387}{-.007\,275} = 0.190\,65$$

$\phi_c' = \underline{190.8°}$

Figure 14.13 shows the complete linkage with the two countermasses added.

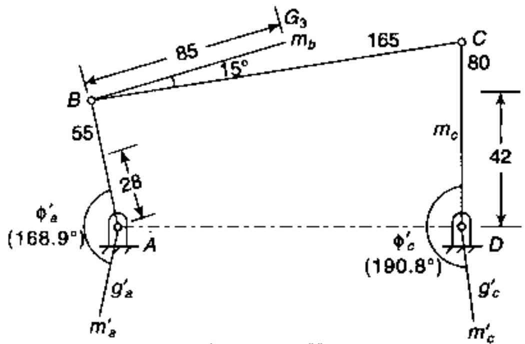


Fig. 14.13

14.6 BALANCING OF RECIPROCATING MASS

Acceleration of the reciprocating mass of a slider-crank mechanism is given by (refer Eq. 13.12)

$$f = r\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

Therefore, the force required to accelerate mass m is

$$F = mr\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

$$= mr\,\omega^2 \cos\theta + mr\omega^2 \frac{\cos 2\theta}{n} \qquad (14.20)$$

$mr\omega^2 \cos\theta$ is called the *primary accelerating force* and $mr\omega^2 \dfrac{\cos 2\theta}{n}$ is called the *secondary accelerating force*.

Maximum value of the primary force = $mr\omega^2$

Maximum value of the secondary force = $\dfrac{mr\omega^2}{n}$

As n is, usually, much greater than unity, the secondary force is small compared with the primary force and can be safely neglected for slow-speed engines.

The inertia force due to primary accelerating force is shown in Fig. 14.14(a). In Fig. 14.14(b), the forces acting on the engine frame due to this inertia force are shown. The force exerted by the crankshaft on the main bearings has two components, $\mathbf{F}_{21}{}^h$ and $\mathbf{F}_{21}{}^v$. The horizontal force $\mathbf{F}_{21}{}^h$ is an unbalanced *shaking force.* The vertical forces $\mathbf{F}_{21}{}^v$ and $\mathbf{F}_{41}{}^v$ balance each other, but form an unbalanced *shaking couple.* The magnitude and direction of this force and couple go on changing with the rotation of the crank angle θ. The shaking force produces linear vibration of the frame in the horizontal direction whereas the shaking couple produces an oscillating vibration.

Thus, it is seen that the shaking

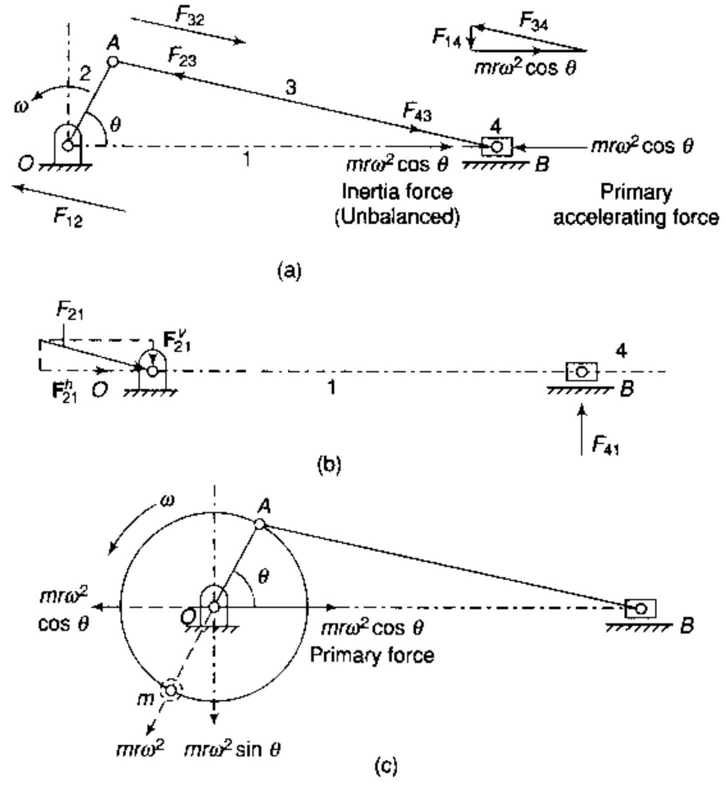


Fig. 14.14

force $\mathbf{F}_{21}{}^h$ is the only unbalanced force. It may hamper the smooth running of the engine and Thus, effort is made to balance the same. However, it is not at all possible to balance it completely and only some modification can be made.

The usual approach of balancing the shaking force is by addition of a rotating countermass at radius r directly opposite the crank which however, provides only a partial balance. This countermass is in addition to the mass used to balance the rotating unbalance due to the mass at the crank pin.

Figure 14.14(c) shows the reciprocating mechanism with a countermass m at the radial distance r. The horizontal component of the centrifugal force due to the balancing mass is $mr\omega^2 \cos\theta$ in the line of stroke. This neutralizes the unbalanced reciprocating force. But the rotating mass also has a component $mr\omega^2 \sin\theta$ perpendicular to the line of stroke which remains unbalanced. The unbalanced force is zero at the ends of the stroke when $\theta = 0°$ or $180°$ and maximum at the middle when $\theta = 90°$. The magnitude or the maximum value of the unbalanced force remains the same, i.e., equal to $mr\omega^2$. Thus, instead of sliding to and fro on its mounting, the mechanism tends to jump up and down.

To minimize the effect of the unbalanced force, a compromise is, usually, made, i.e., 2/3 of the reciprocating mass is balanced (or a value between one-half and three-quarters). If c is the fraction of the reciprocating mass Thus, balanced then

primary force balanced by the mass $= cmr\omega^2 \cos\theta$

primary force unbalanced by the mass $= (1 - c)cmr\omega^2 \cos\theta$

vertical component of centrifugal force which remains unbalanced

$$= cmr\omega^2 \sin\theta$$

In fact, in reciprocating engines, unbalanced forces in the direction of the line of stroke are more dangerous than the forces perpendicular to the line of stroke.

Resultant unbalanced force at any instant

$$= \sqrt{[(1-c)\,mr\omega^2 \cos\theta]^2 + [cmr\omega^2 \sin\theta]^2} \tag{14.21}$$

The resultant unbalanced force is minimum when $c = 1/2$.

The method just discussed above to balance the disturbing effect of a reciprocating mass is just equivalent to as if a revolving mass at the crankpin is completely balanced by providing a countermass at the same radius diametrically opposite the crank. Thus, if m_p is the mass at the crankpin and c is the fraction of the reciprocating mass m to be balanced, the mass at the crankpin may be considered as $(c_m + m_p)$ which is to be completely balanced.

Example 14.9 *The following data relate to a single-cylinder reciprocating engine:*

Mass of reciprocating parts = 40 kg

Mass of revolving parts = 30 kg at crank radius

Speed = 150 rpm

Stroke = 350 mm

If 60% of the reciprocating parts and all the revolving parts are to be balanced, determine the

(i) balance mass required at a radius of 320 mm

(ii) unbalanced force when the crank has turned 45° from the top-dead centre

Solution

$$\omega = \frac{2\pi \times 150}{60} = 15.7 \text{ rad/s}$$

$$r = \frac{350}{2} = 175 \text{ mm}$$

(i) Mass to be balanced at the crankpin $= cm + m_p$

$= 0.6 \times 40 + 30$

$= 54$ kg

$m_c r_c = mr$

$m_c \times 320 = 54 \times 175$

$m_c = 29.53$ kg

(ii) Unbalanced force (at $\theta = 45°$)

$$= \sqrt{[(1-c)\,mr\omega^2 \cos\theta]^2 + (cmr\omega^2 \sin\theta)^2}$$

$$= \sqrt{\begin{array}{l}[(1-0.6)\times 40\times 0.175\times (15.7)^2 \cos 45°]^2 \\ +[0.6\times 40\times 0.175\times (15.7)^2 \sin 45°]^2\end{array}}$$

$= \underline{880.7\text{ N}}$

14.7 BALANCING OF LOCOMOTIVES

Locomotives are of two types, coupled and uncoupled. If two or more pairs of wheels are coupled together to increase the adhesive force between the wheels and the track, it is called a coupled locomotive. Otherwise, it is an uncoupled locomotive.

Locomotives usually have two cylinders. If the cylinders are mounted between the wheels, it is called an inside cylinder locomotive and if the cylinders are outside the wheels, it is an outside cylinder locomotive. The cranks of the two cylinders are set at 90° to each other so that the engine can be started easily after stopping in any position. Balance masses are placed on the wheels in both types.

In coupled locomotives, wheels are coupled by connecting their crankpins with coupling rods. As the coupling rod revolves with the crankpin, its proportionate mass can be considered as a revolving mass which can be completely balanced.

Thus, whereas in uncoupled locomotives, there are four planes for consideration, two of the cylinders and two of the driving wheels, in coupled locomotives there are six planes, two of cylinders, two of coupling rods and two of the wheels. The planes which contain the coupling rod masses lie outside the planes that contain the balance (counter) masses. Also, in case of coupled locomotives, the mass required to balance the reciprocating parts is distributed among all the wheels which are coupled. This results in a reduced hammer-blow (refer Sec. 14.8).

Locomotives have become obsolete nowadays.

14.8 EFFECTS OF PARTIAL BALANCING IN LOCOMOTIVES

1. Hammer-blow

Hammer-blow is the maximum vertical unbalanced force caused by the mass provided to balance the reciprocating masses. Its value is $mr\omega^2$. Thus, it varies as a square of the speed. At high speeds, the force of the hammer-blow could exceed the static load on the wheels and the wheels can be lifted off the rail when the direction of the hammer-blow will be vertically upwards.

2. Variation of Tractive Force

A variation in the tractive force (effort) of an engine is caused by the unbalanced portion of the primary force which acts along the line of stroke of a locomotive engine.

If c is the fraction of the reciprocating mass that is balanced then

unbalanced primary force for cylinder 1 $= (1-c)\,mr\omega^2 \cos\theta$

unbalanced primary force for cylinder 2 $= (1-c)\,mr\omega^2 \cos(90° + \theta)$

$= -(1-c)\,mr\omega^2 \sin\theta$

Total unbalanced primary force or the variation in the tractive force

$= -(1-c)\,mr\omega^2 (\cos\theta - \sin\theta)$

This is maximum when $(\cos\theta - \sin\theta)$ is maximum, or when $\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0$

or $-\sin\theta - \cos\theta = 0$

or $\sin\theta = -\cos\theta$

or $\tan\theta = -1$

or $\theta = 135°$ or $315°$

When $\theta = 135°$

maximum variation in tractive force

$$= (1-c)mr\omega^2(\cos 135° - \sin 135°)$$

$$= (1-c)mr\omega^2\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$$= -\sqrt{2}\,(1-c)mr\omega^2$$

When $\theta = 315°$

Maximum variation in tractive force

$$= (1-c)mr\omega^2(\cos 315° - \sin 315°)$$

$$= (1-c)mr\omega^2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2}\,(1-c)mr\omega^2$$

Thus, maximum variation

$$= \pm\sqrt{2}\,(1-c)mr\omega^2 \qquad (14.22)$$

3. Swaying Couple

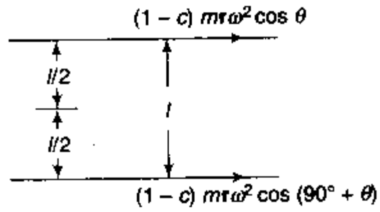


Fig. 14.15

Unbalanced primary forces along the lines of stroke are separated by a distance l apart and Thus, constitute a couple (Fig. 14.15). This tends to make the leading wheels sway from side to side.

Swaying couple = moments of forces about the engine centre line

$$= [(1-c)\,mr\omega^2\cos\theta]\frac{l}{2} - [(1-c)\,mr\omega^2\cos(90° + \theta)]\frac{l}{2}$$

$$= (1-c)\,mr\omega^2(\cos\theta + \sin\theta)\,\frac{l}{2}$$

This is maximum when $(\cos\theta + \sin\theta)$ is maximum.

i.e., when $\frac{d}{dt}(\cos\theta + \sin\theta) = 0$

or $-\sin\theta + \cos\theta = 0$

or $\sin\theta = \cos\theta$

or $\tan\theta = 1$

or $\theta = 45°$ or $225°$

When $\theta = 45°$, maximum swaying couple $= \frac{1}{\sqrt{2}}(1-c)\,mr\omega^2 l$

When $\theta = 225°$, maximum swaying couple $= -\frac{1}{\sqrt{2}}(1-c)\,mr\omega^2 l$

Thus, maximum swaying couple $= \pm\frac{1}{\sqrt{2}}(1-c)mr\omega^2 l$ (14.23)

Example 14.10 *The following data refer to a two-cylinder uncoupled locomotive:*

Rotating mass per cylinder	*= 280 kg*
Reciprocating mass per cylinder	*= 300 kg*
Distance between wheels	*= 1400 mm*
Distance between cylinder centres	*= 600 mm*
Diameter of treads of driving wheels	*= 1800 mm*
Crank radius	*= 300 mm]*
Radius of centre of balance mass	*= 620 mm*
Locomotive speed	*= 50 km/hr*
Angle between cylinder cranks	*= 90°*
Dead load on each wheel	*= 3.5 tonne*

Determine the

(i) balancing mass required in the planes of driving wheels if whole of the revolving and two-third of the reciprocating mass are to be balanced
(ii) swaying couple
(iii) variation in the tractive force
(iv) maximum and minimum pressure on the rails
(v) maximum speed of locomotive without lifting the wheels from the rails

Solution Total mass to be balanced $= m_p + cm$

$$= 280 + \frac{2}{3} \times 300$$

$$= 480 \text{ kg}$$

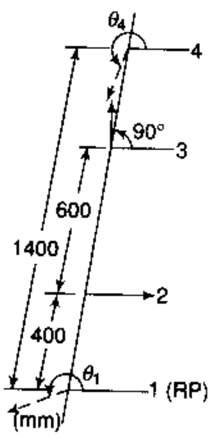


Fig. 14.16

(i) Take 1 as the reference plane and angle $\theta_2 = 0°$ (Fig. 14.16). Writing the couple equations,

$$m_2 \mathbf{r}_2 l_2 \cos\theta_2 + m_3 \mathbf{r}_3 l_3 \cos\theta_3 + m_4 \mathbf{r}_4 l_4 \cos\theta_4 = 0$$

or $480 \times 300 \times 400 \cos 0° + 480 \times 300 \times 1000 \cos 90° + m_4 \times 620 \times 1400 \cos\theta_4 = 0$

or $m_4 \cos\theta_4 = -66.36$ (i)

and $m_2 \mathbf{r}_2 l_2 \sin\theta_2 + m_3 \mathbf{r}_3 l_3 \sin\theta_3 + m_4 \mathbf{r}_4 l_4 \sin\theta_4 = 0$

or $480 \times 300 \times 400 \sin 0° + 480 \times 300 \times 1000 \sin 90° + m_4 \times 620 \times 1400 \sin\theta_4 = 0$

or

$m_4 \sin\theta_4 = -165.9$ (ii)

Squaring and adding (i) and (ii), $m_4 = \underline{178.7 \text{ kg}}$

Dividing (ii) by (i), $\tan\theta_4 = \frac{-165.9}{-66.36} = 2.5$

$$\theta_4 = \underline{248.2°}$$

Taking 4 as the reference plane and writing the couple equations,

$$m_2 \mathbf{r}_2 l_2 \cos\theta_2 + m_3 \mathbf{r}_3 l_3 \cos\theta_3 + m_1 \mathbf{r}_1 l_1 \cos\theta_1 = 0$$

$480 \times 300 \times 1000 \cos 0° + 480 \times 300 \times 400 \cos 90° + m_1 \times 620 \times 1400 \sin\theta_1 = 0$

or

$m_1 \sin\theta_1 = -165.9$ (iii)

Similarly, $m_1 \sin\theta_1 = -66.36$ (iv)

From (iii) and (iv), $m_1 = \underline{178.7 \text{ kg}} = m_4$

$\tan\theta_1 = \frac{-66.36}{-165.9} = 0.4$ or $\theta_1 = \underline{201.8°}$

The treatment shows that the magnitude of m_1 could have directly been written equal to m_4.

(ii) $\omega = \frac{50 \times 1000 \times 1000}{60 \times 60} \times \frac{1}{\frac{1800}{2}} = 15.43$ rad/s

$$\text{Swaying couple} = \pm \frac{1}{\sqrt{2}}(1 - c)\, m r \omega^2 l$$

$$= \pm \frac{1}{\sqrt{2}}\left(1 - \frac{2}{3}\right) \times 300 \times 0.3 \times (15.43)^2 \times 0.6$$

$$= \underline{3030.3 \text{ N.m}}$$

(iii) Variation in tractive force $= \pm\sqrt{2}(1 - c)\, m r \omega^2$

$$= \pm\sqrt{2}\left(1 - \frac{2}{3}\right) \times 300 \times 0.3 \times (15.43)^2$$

$$= \underline{10\,100 \text{ N}}$$

(iv) Balance mass for reciprocating parts only

$$= 178.7 \times \frac{\frac{2}{3} \times 300}{480} = 74.46 \text{ kg}$$

Hammer-blow $= mr\omega^2$

$= 74.46 \times 0.62 \times (15.43)^2$

$= 10\,991$ N

Dead load $= 3.5 \times 1000 \times 9.81 = 34\,335$ N

Maximum pressure on rails

$= 34\,335 + 10\,991 = \underline{45\,326 \text{ N}}$

Minimum pressure on rails

$= 64\,335 - 10\,991 = \underline{23\,344 \text{ N}}$

(v) Maximum speed of the locomotive without lifting the wheels from the rails will be when the dead load becomes equal to the hammer-blow.

i.e., $74.46 \times 0.62 \times \omega^2 = 34\,335$

or $\omega = 27.27$ rad/s

Velocity of wheels

$$= \omega r = \left(27.27 \times \frac{1.80}{2}\right) \text{m/s}$$

$$= \left(27.27 \times \frac{1.8}{2} \times \frac{60 \times 60}{1000}\right) \text{km/h}$$

$= 88.36$ km/h

Example 14.11 *The following data refer to a four-coupled wheel locomotive with two inside cylinders:*

Pitch of cylinders	=	*600 mm*
Reciprocating mass/cylinder	=	*315 kg*
Revolving mass/cylinder	=	*260 kg*
Distance between driving wheels	=	*1.6 m*
Distance between coupling rods	=	*2 m*
Diameter of driving wheels	=	*1.9 m*
Revolving parts for each coupling rod crank	=	*130 kg*
Engine crank radius	=	*300 mm*
Coupling rod crank radius	=	*240 mm*
Distance of centre of balance mass in planes of Driving wheels from axle centre	=	*750 mm*
Angle between engine cranks	=	*90°*
Angle between coupling rod crank with adjacent engine crank	=	*180°*

The balanced mass required for the reciprocating parts is equally divided between each pair of coupled wheels. Determine the

(i) magnitude and position of the balance mass required to balance two-third of reciprocating and whole of the revolving parts

(ii) hammer-blow and the maximum variation of tractive force when the locomotive speed is 80 km/h

Solution *Leading wheels* Balance mass on each leading wheel

$$= m_p + \frac{1}{2} cm$$

$$= 260 + \frac{1}{2}\left(\frac{2}{3} \times 315\right)$$

$= 365$ kg

Taking the plane 2 as the reference plane and $\angle \theta_3 = 0°$ [refer Fig.14.17]

$m_1 = m_6 = 130$ kg; $m_3 = m_4 = 365$ kg

$r_1 = r_6 = 0.24$ m; $r_2 = r_5 = 0.75$ m; $r_3 = r_4 = 0.3$ m

$l_1 = -0.2$ m; $l_3 = 0.5$ m; $l_4 = 1.1$ m; $l_5 = 1.6$ m; $l_6 = 1.8$ m

$m_1 r_1 l_1 = 130 \times 0.24 \times (-0.2) = -6.24$

$m_3 r_3 l_3 = 365 \times 0.3 \times 0.5 = 54.75$

$m_4 r_4 l_4 = 365 \times 0.3 \times 1.1 = 120.45$

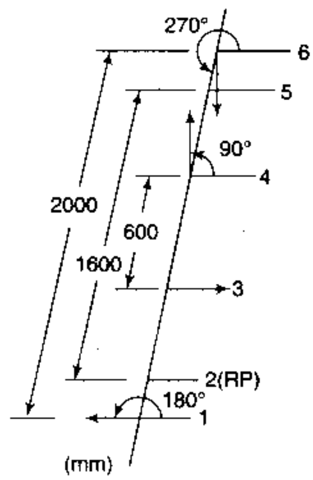


Fig. 14.17

$m_5 r_5 l_5 = m_5 \times 0.75 \times 1.6 = 1.2\, m_5$
$m_6 r_6 l_6 = 130 \times 0.24 \times 1.8 = 56.16$

$$1.2m_5 = \begin{bmatrix} (-6.24\cos 180° + 54.75\cos 0° \\ +120.45\cos 90° + 56.16\cos 270°)^2 \\ +(-6.24\sin 180° + 54.75\sin 0° + 120.45 \\ \sin 90° + 56.16\sin 270°)^2 \end{bmatrix}^{1/2}$$

$= [(60.99)^2 + (64.29)^2]^{1/2}$
$= 88.62$

$m_5 = \underline{73.85 \text{ kg}}$

$$\tan\theta_5 = \frac{-64.29}{-60.99} = 1.054 \text{ or } \theta_5 = \underline{226.5°}$$

From symmetry of the system, $m_2 = m_5$ $\underline{73.85 \text{ kg}}$

$$\text{and } \tan\theta_2 = \frac{-60.99}{-64.29} = 0.949 \text{ or } \theta_2 = \underline{223.5°}$$

Trailing Wheels The arrangement remains the same except that only half of the required reciprocating masses have to be balanced at the cranks.

i.e., $m_3 = m_4 = \frac{1}{2}\left(\frac{2}{3} \times 315\right) = 105 \text{ kg}$

Then, $m_3 r_3 l_3 = 105 \times 0.3 \times 0.5 = 15.75$
and $m_4 r_4 l_4 = 105 \times 0.3 \times 1.1 = 34.65$

$$1.2m_5 = \begin{bmatrix} (-6.24\cos 180° + 15.75\cos 0° \\ +34.65\cos 90° + 56.16\cos 270°)^2 \\ +(-6.24\sin 180° + 15.75\sin 0° + 34.65 \\ \sin 90° + 56.16\sin 270°)^2 \end{bmatrix}^{1/2}$$

$= [(21.99)^2 + (-21.51)^2]^{1/2}$
$= 30.76$

$m_5 = 25.63 \text{ kg}$

$$\tan\theta_5 = \frac{-(-21.51)}{-21.99} = \frac{+21.51}{-21.99} = -0.978$$

or $\theta_5 = \underline{135.4°}$

By symmetry, $m_2 = m_5 = \underline{25.63 \text{ kg}}$

$$\text{and } \tan\theta_2 = \frac{-21.99}{+21.51} = -1.022 \text{ or } \theta_2 = \underline{314.4°}$$

(ii) Hammer–blow = $mr\omega$
where m is the balance mass for reciprocating parts only and neglecting m_1 and m_6 in the above calculations.
Thus, $m_1 r_1 l_1 = m_6 r_6 l_6 = 0$

$$1.2m_5 = \begin{bmatrix} (15.75\cos 0° + 34.65\cos 90°)^2 \\ +(15.75\sin 0° + 34.65\sin 90°) \end{bmatrix}^{1/2}$$

$= [(15.75)^2 + (34.65)^2]^{1/2}$
$= 38.06$

$m_5 = 31.75 \text{ kg}$

$$\omega = \frac{80 \times 1000}{60 \times 60} \times \frac{1}{1.9/2} = 23.39 \text{ rad/s}$$

Hammer-blow = $31.72 \times 0.75 \times (23.39)^2 = 13\,015$ N
Maximum variation of tractive force

$= \pm\sqrt{2}(1-c)mr\omega^2$

$= \pm\sqrt{2}\left(1-\frac{2}{3}\right) \times 315 \times 0.3 \times (23.39)^2$

$= \underline{\pm\ 24\ 372 \text{ N}}$

14.9 SECONDARY BALANCING

It was stated earlier that the secondary acceleration force is defined as

$$\text{secondary force} = mr\omega^2 \frac{\cos 2\theta}{n} \tag{14.24}$$

Its frequency is twice that of the primary force and the magnitude $1/n$ times the magnitude of the primary force.

The expression can also be written as $mr\,(2\omega)^2 \dfrac{\cos 2\theta}{4n}$

Now, consider two cranks of an engine (Fig. 14.18). One actual one and the other imaginary, with the following specifications:

	Actual	Imaginary
Angular velocity	ω	2ω
Length of crank	r	$\frac{r}{4n}$
Mass at the crank pin	m	m

Thus, when the actual crank has turned through an angle $\theta = \omega t$, the imaginary crank would have turned an angle of $2\theta = 2\omega t$

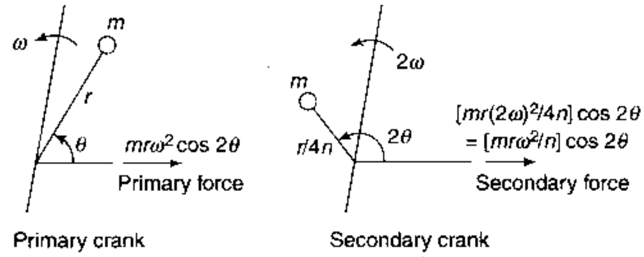


Fig. 14.18

$$\text{Centrifugal force induced in the imaginary crank} = \frac{mr(2\omega)^2}{4n}$$

$$\text{Component of this force along line of stroke} = \frac{mr(2\omega)^2}{4n} \cos 2\theta$$

Thus, the effect of the secondary force is equivalent to an imaginary crank of length $r/4n$ rotating at double the angular velocity, i.e., twice of the engine speed.

The imaginary crank coincides with the actual at inner top-dead centre. At other times, it makes an angle with the line of stroke equal to twice that of the engine crank.

The secondary couple about a reference plane is given by the multiplication of the secondary force with the distance l of the plane from the reference plane.

Complete Balancing of Reciprocating Parts

From the foregoing discussion, it is concluded that for complete balancing of the reciprocating parts, the following conditions must be fulfilled:

1. Primary forces must balance, i.e., primary force polygon is enclosed.
2. Primary couples must balance, i.e., primary couple polygon is enclosed.
3. Secondary forces must balance, i.e., secondary forces polygon is enclosed.
4. Secondary couples must balance, i.e., secondary couple polygon is enclosed.

Usually, it is not possible to satisfy all the above conditions fully for a multicylinder engine. Mostly some unbalanced force or couple would exist in the reciprocating engines.

14.10 BALANCING OF INLINE ENGINES

If a reciprocating mass is transferred to the crankpin, the axial component parallel to the cylinder axis of the resulting centrifugal force represents the primary unbalanced force.

Consider a shaft (Fig. 14.19) consisting of three equal cranks unsymmetrically spaced. The crankpins carry equivalents of three unequal reciprocating masses. Then

$$\text{Primary force} = \Sigma mr\omega^2 \cos\theta \qquad (14.25)$$

$$\text{Primary couple} = \Sigma mr\omega^2\, l \cos\theta \qquad (14.26)$$

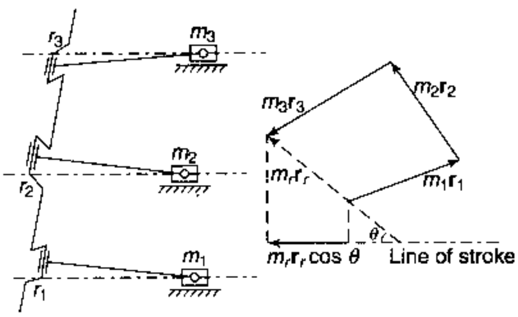


Fig. 14.19

$$\text{Secondary force} = \Sigma\, mr\frac{(2\omega)^2}{4n}\cos 2\theta = \Sigma\, mr\frac{\omega^2}{n}\cos 2\theta \tag{14.27}$$

$$\text{Secondary couple} = \Sigma\, mr\frac{(2\omega)^2}{4n}l\cos 2\theta = \Sigma\, mr\frac{\omega^2}{n}l\cos 2\theta \tag{14.28}$$

In order to solve the above equations graphically, first draw the $\Sigma mr\cos\theta$ polygon (ω^2 is common to all forces). Then the axial component of the resultant force ($\mathbf{F}_r\cos\theta$) multiplied by ω^2 provides the primary unbalanced force on the system at that moment. This unbalanced force is zero when $\theta = 90°$ and a maximum when $\theta = 0°$.

In case the force polygon encloses, the resultant as wall as the axial component will always be zero and Thus, the system will be in primary balance. Then $\Sigma\mathbf{F}_{ph} = 0$ and $\Sigma\mathbf{F}_{pv} = 0$.

To find the secondary unbalance force, first find the positions of the imaginary secondary cranks. Then transfer the reciprocating masses and multiply the same by $(2\omega)^2/4n$ or ω^2/n to get the secondary force.

In the same way primary and secondary couple (mrl) polygons can be drawn for primary and secondary couples.

In the following paragraphs, some multi-crank arrangements have been examined.

1. In-line Two-cylinder Engine

Consider a two-cylinder engine (Fig. 14.20), cranks of which are 180° apart and have equal reciprocating masses. Taking a plane through the centre line as the reference plane,

Primary force = $mr\omega^2\,[\cos\theta + \cos(180° + \theta)] = 0$

$$\text{Primary couple} = mr\omega^2\left[\frac{l}{2}\cos\theta + \left(-\frac{l}{2}\right)\cos(180° + \theta)\right] = mr\omega^2 l\cos\theta$$

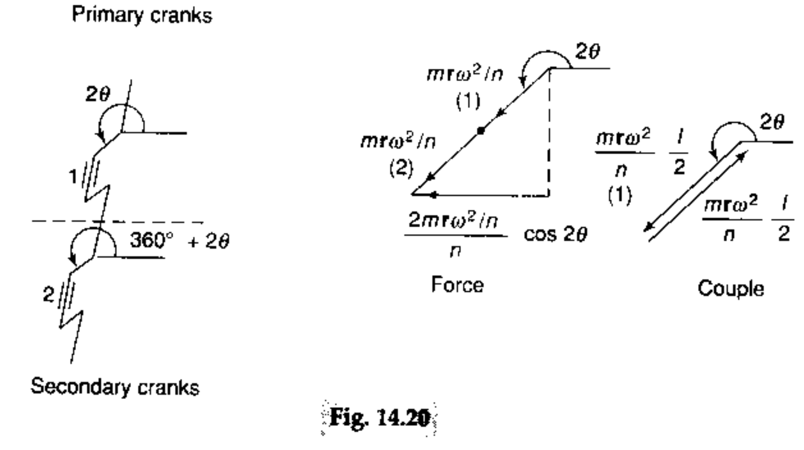


Fig. 14.20

Maximum values are $mr\omega^2 l$ at $\theta = 0°$ and $180°$

$$\text{Secondary force} = \frac{mr\omega^2}{n}[\cos 2\theta + \cos(360° + 2\theta)] = 2\frac{mr\omega^2}{n}\cos 2\theta$$

Maximum values are $\frac{2mr\omega^2}{n}$ when $2\theta = 0°, 180°, 360°$ and $540°$

or $\theta = 0°, 90°, 180°$, and $270°$

$$\text{Secondary couple} = \frac{mr\omega^2}{n}\left[\frac{l}{2}\cos 2\theta + \left(-\frac{l}{2}\right)\cos(360° + 2\theta)\right] = 0$$

Remember that to find the primary forces and couples analytically, the positions of the cranks have to be taken in terms of θ. As it is a rotating system, the maximum values or magnitudes of these forces and couples vary instant to instant and are equal to the values as given by the equivalent rotating masses at the crank pin. If a particular position of the crankshaft is considered, the above expressions may not give the maximum value. For example, the maximum value of primary couple in this case is found to be $mr\omega^2.l$. This is the value which is obtained when the crank positions are $0°$ and $180°$. However, if the crank positions are assumed at $90°$ and $270°$, the values obtained are zero. Thus, in case any particular position of the crankshaft is considered, then both x- and y-components of the force and couple can be taken to find the maximum values, e.g., if the positions of the cranks are considered at $120°$ and $300°$, the primary couple can be obtained as below:

$$x\text{-component} = mr\omega^2\left[\frac{l}{2}\cos 120° + \left(-\frac{l}{2}\right)\cos(180° + 120°)\right] = -\frac{1}{2}mr\omega^2 l$$

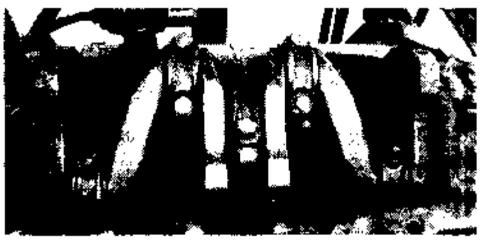
$$y\text{-component} = mr\omega^2\left[\frac{l}{2}\sin 120° + \left(-\frac{l}{2}\right)\sin(180° + 120°)\right] = \frac{\sqrt{3}}{2}mr\omega^2 l$$

$$\text{Primary couple} = \sqrt{\left(-\frac{1}{2}mr\omega^2 l\right)^2 + \left(\frac{\sqrt{3}}{2}mr\omega^2 l\right)^2} = mr\omega^2 l$$

The graphical solution has also been shown in Fig. 14.20 which is self-explanatory.

2. In-line Four-cylinder Four-stroke Engine

Such an engine has two outer as well as inner cranks (throws) in line. The inner throws are at $180°$ to the outer throws. Thus the angular positions for the cranks are θ for the first, $(180° + \theta)$ for the second, $(180° + \theta)$ for the third, and θ for the fourth (Fig. 14.21).



Crankshaft of a four-cylinder engine

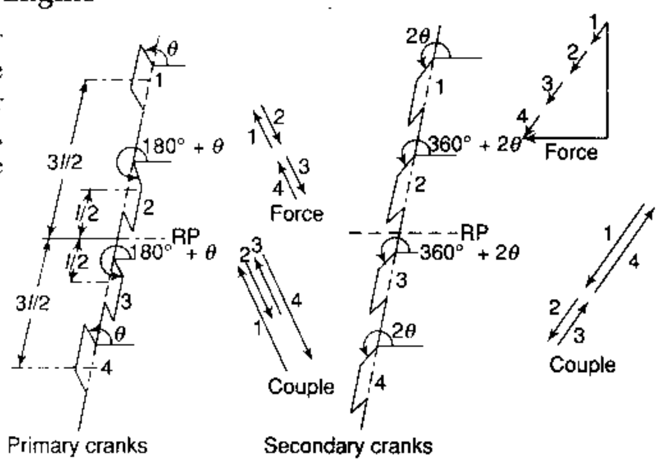


Fig. 14.21

For convenience, choose a plane passing through the middle bearing about which the arrangement is symmetrical as the reference plane.

Primary force

$= mr\omega^2 [\cos\theta + \cos(180° + \theta) + \cos(180° + \theta) + \cos\theta] = 0$

Primary couple

$$= mr\omega^2 \left[\frac{3l}{2}\cos\theta + \frac{l}{2}\cos(180° + \theta) + \left(-\frac{l}{2}\right)\cos(180° + \theta) + \left(-\frac{3l}{2}\right)\cos\theta\right]$$

$= 0$

Secondary force

$$= \frac{mr\omega^2}{n}[\cos 2\theta + \cos(360° + 2\theta) + \cos(360° + 2\theta) + \cos 2\theta]$$

$$= \frac{4mr\omega^2}{n}\cos 2\theta$$

Maximum value $= \dfrac{4mr\omega^2}{n}$ at $2\theta = 0°, 180°, 360°$ and $540°$ or $\theta = 0°, 90°, 180°$ and $270°$

Secondary couple

$$= \frac{mr\omega^2}{n}\left[\frac{3l}{2}\cos 2\theta + \frac{l}{2}\cos(360° + 2\theta) + \left(-\frac{l}{2}\right)\cos(360° + 2\theta) + \left(-\frac{3l}{2}\right)\cos 2\theta\right]$$

$= 0$

Graphical solution has been shown in Fig. 14.21. Thus this engine is not balanced in secondary forces.

3. Six-cylinder Four-stroke Engine

Only a graphical solution is being given for simplicity. In a four-stroke engine, the cycle is completed in two revolutions of the crank and the cranks are 120° apart.

Crank positions for different cylinders for the firing order 142635 for clockwise rotation of the crankshaft are

For first, $\theta = 0°$ For fourth, $\theta = 120°$
For second, $\theta = 240°$ For fifth, $\theta = 240°$
For third, $\theta = 120°$ For sixth, $\theta = 0°$

Assuming m and r equal for all cylinders and taking a vertical plane passing through the middle of the shaft as the reference plane, the force and the couple polygons are drawn as shown in Fig. 14.22.

Since all the force and couple polygons close, it is an inherently balanced engine for primary and secondary forces and couples.

Crankshaft of a six-cylinder engine

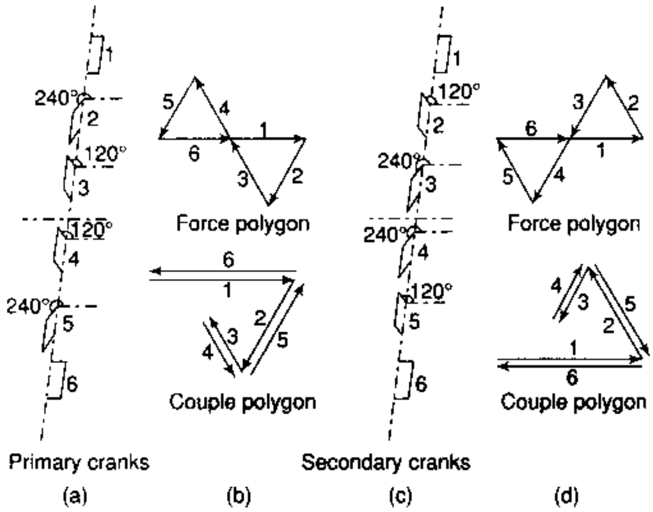


Fig. 14.22

Example 14.12 *A four-cylinder oil engine is in complete primary balance. The arrangement of the reciprocating masses in different planes is as shown in Fig.14.23(a). The stroke of each piston is 2r mm. Determine the reciprocating mass of the cylinder 2 and the relative crank positions.*

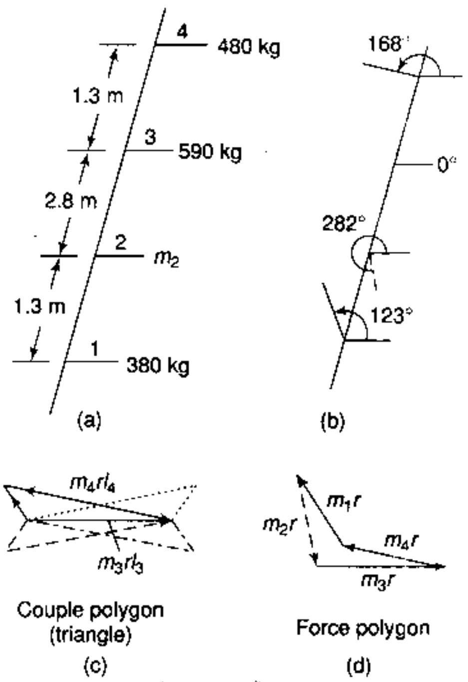


Fig. 14.23

Solution Crank length = $2r/2 = r$

Take 2 as the reference plane and $\theta_3 = 0°$

$m_1r_1l_1 = 380\,r \times (-1.3) = -494r$ $\quad m_1r_1 = 380\,r$

$m_3r_3l_3 = 590\,r \times 2.8 = 1652r$ $\quad m_3r_3 = 590\,r$

$m_4r_4l_4 = 480\,r \times (2.8 + 1.3) = 1968r$ $\quad m_4r_4 = 480\,r$

$-494\,r\cos\theta_1 + 1652\,r\cos 0° + 1968\,r\cos\theta_4 = 0$

or $\quad 494\cos\theta_1 = 1652 + 1968\cos\theta_4$ (i)

and $\quad -494r\sin\theta_1 + 1652r\sin 0° + 1668r\sin\theta_4 = 0$

or $\quad 494\sin\theta_1 = 1968\sin\theta_4$ (ii)

Squaring and adding (i) and (ii),

$(494)^2 = (1652 + 1968\cos\theta_4)^2 + (1968\sin\theta_4)^2$

$= (1652)^2 + (1968)^2\cos^2\theta_4 + 2 \times 1652 \times 1968\cos\theta_4 + (1968)^2\sin^2\theta_4$

$= (1652)^2 + (1968)^2 + 2 \times 1652 \times 1968\cos\theta_4$

$\cos\theta_4 = -0.978$

or $\theta_4 = 167.9°$ or $192.1°$

Choosing one value, say $\theta_4 = \underline{167.9°}$

Dividing (ii) by (i), $\tan\theta_1 = \dfrac{1968\sin 167.9°}{1652 + 1968\cos 167.9°}$

$= \dfrac{+412.53}{-272.28}$

$= -1.515$

$\theta_1 = 123.4°$

Writing the force equation, (r is common),

$380\cos 123.4° + m_2\cos\theta_2 + 590\cos 0° + 480\cos 167.9° = 0$

or $\quad m_2\cos\theta_2 = 88.5$ (iii)

and $\quad 380\sin 123.4° + m_2\sin\theta_2 + 590\sin 0° + 480\sin 167.9° = 0$

or $\quad m_2\sin\theta_2 = -417.9$ (iv)

Squaring and adding (iii) and (iv), $m_2 = \underline{427.1\text{ kg}}$

Dividing (iii) by (iv), $\tan\theta_2 = \dfrac{-417.9}{+88.5} = -4.72$

or $\quad \theta_2 = \underline{282°}$

Figure 14.23(b) shows the relative crank positions.

Had we chosen $\theta_4 = 192.1°$, a different set of values of m_2, θ_1 and θ_2 would have come.

To solve the problem graphically, draw the couple polygon (triangle) as shown in Fig. 14.23(c) from the three known values. This provides the relative direction of the masses m_1, m_3 and m_4. Now, complete the force polygon [Fig. 14.23(d) and obtain the magnitude and direction of m_2. The results obtained are $\theta_4 = 168°$, $\theta_1 = 123°$, $\theta_2 = 282°$.

Also, $m_2r = 427r$ or $m_2 = 427$ kg

Note that the couple triangle can be drawn in more than one way. However, only two sets of answers are obtained. Also, $m_1\mathbf{r}_1l_1$ is negative and, therefore, its direction is reversed in the diagram.

Example 14.13 *The arrangement of the cranks of a 4-crank symmetrical engine is shown in Fig.14.24. The reciprocating masses at cranks 1 and 4 are each equal to m_1 and of the cranks 2 and 3 are each equal to m_2. Show that the arrangement is balanced for primary forces and couples and for secondary forces if*

$$\frac{m_1}{m_2} = \frac{\cos\beta}{\cos\alpha},\ \frac{l_1}{l_2} = \frac{\tan\beta}{\tan\alpha},\ \cos\alpha\cos\beta = \frac{1}{2}$$

Determine also the magnitude of the out-of balance secondary couple if the system rotates at ω rad/s.

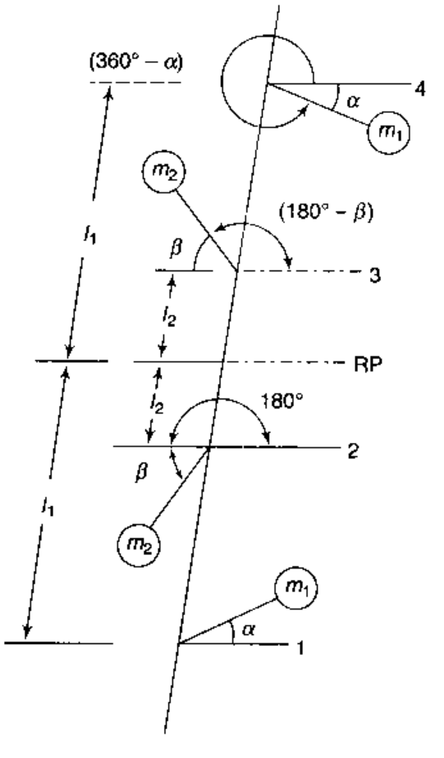


Fig. 14.24

Solution As particular positions of the cranks are being considered, horizontal and vertical components of primary and secondary forces and couples must be taken.

(i) Primary Forces

$$\Sigma F_{ph} = r\omega^2 \begin{bmatrix} m_1 \cos\alpha + m_2 \cos(180° + \beta) \\ +m_2 \cos(180° - \beta) \\ +m_1 \cos(180° - \alpha) \end{bmatrix}$$

$$= 2r\,\omega^2\,[m_1 \cos\alpha - m_2 \cos\beta]$$

$$\Sigma F_{pv} = r\omega^2 \begin{bmatrix} m_1 \sin\alpha + m_2 \sin(180° + \beta) + m_2 \\ \sin(180° - \beta) + m_2 \sin(360° - \alpha) \end{bmatrix}$$

$$= 0$$

For primary balance of forces, $\Sigma \mathbf{F}_{ph}$ must be zero,

i.e., $m_1 \cos\alpha - m_2 \cos\beta = 0$

$$\text{or} \quad \frac{m_1}{m_2} = \frac{\cos\beta}{\cos\alpha}$$

(ii) Primary Couples Take reference plane at the middle of shaft about which the system is symmetrical.

$$\Sigma C_{ph} = r\omega^2 \begin{bmatrix} m_1(-l_i)\cos\alpha + m_2(-l_2) \\ \cos(180° + \beta) + m_2(l_2) \\ \cos(180° - \beta) + m_1(l_1) \\ \cos(360° - \alpha) \end{bmatrix}$$

$$= r\,\omega^2\,[-m_1 l_1 \cos\alpha + m_2 l_2 \cos\beta - m_2 l_2 \cos\beta + m_1 l_1 \cos\alpha]$$

$$= 0$$

$$C_{pv} = r\omega^2 \begin{bmatrix} m_1(-l_i)\sin\alpha + m_2(-l_2) \\ \sin(180° + \beta) + m_2(-l_2) \\ \sin(180° - \beta) + m_1(-l_1) \\ \sin(360° - \alpha) \end{bmatrix}$$

$$= 2r\,\omega^2\,[m_2 l_2 \sin\beta - m_1 l_1 \sin\alpha]$$

Thus for balancing of primary couples,

$$m_2 l_2 \sin\beta - m_1 l_1 \sin\alpha = 0$$

$$\text{or} \quad \frac{l_1}{l_2} = \frac{m_2 \sin\beta}{m_1 \sin\alpha} = \frac{\cos\alpha \sin\beta}{\cos\beta \sin\alpha} = \frac{\tan\beta}{\tan\alpha}$$

(iii) Secondary Forces

$$\Sigma F_{sh} = \frac{r\omega^2}{n} \begin{bmatrix} m_1 \cos 2\alpha + m_2 \cos 2(180° + \beta) \\ + m_2 \cos 2(180° - \beta) \\ + m_1 \cos 2(360° - \alpha) \end{bmatrix}$$

$$= \frac{r\omega^2}{n}[m_1 \cos 2\alpha + m_2 \cos 2\beta + m_2 \cos 2\beta + m_1 \cos 2\alpha]$$

$$= \frac{2r\omega^2}{n}[m_1 \cos 2\alpha + m_2 \cos 2\beta]$$

$$\Sigma F_{sv} = \frac{r\omega^2}{n} \begin{bmatrix} m_1 \sin 2\alpha + m_2 \sin 2(180° + \beta) + m_2 \\ \sin 2(180° - \beta) + m_1 \sin 2(360° - \alpha) \end{bmatrix}$$

$$= \frac{r\omega^2}{n}[m_1 \sin 2\alpha + m_2 \sin 2\beta - m_2 \sin 2\beta - m_1 \sin 2\alpha]$$

$$= 0$$

For the balancing of secondary forces,

$$m_1 \cos 2\alpha + m_2 \cos 2\beta = 0$$

or $\frac{m_1}{m_2} \cos \alpha + \cos 2\beta = 0$

or $\frac{\cos\beta}{\cos\alpha}(2\cos^2\alpha - 1) + (2\cos^2\beta - 1) = 0$

or $2\cos\beta\cos^2\alpha = \cos\beta + 2\cos^2\beta\cos\alpha - \cos\alpha = 0$

or $2\cos\beta\cos\alpha(\cos\alpha + \cos\beta) - (\cos\alpha + \cos\beta) = 0$

or $(2\cos\alpha + \cos\beta)(2\cos\beta\cos\alpha - 1) = 0$

As $\cos\alpha + \cos\beta \neq 0$,

$\therefore\ 2\cos\beta\cos\alpha - 1 = 0$

or $\cos\alpha\cos\beta = \frac{1}{2}$

(iv) Secondary Couples

$$\sum C_{sh} = \frac{r\omega^2}{n}\begin{bmatrix} m_1(-l_i)\cos 2\alpha + m_2(-l_2) \\ \cos 2(180° + \beta) + m_2(l_2) \\ \cos 2(180° - \beta) + m_1(l_1) \\ \cos 2(360° - \alpha) \end{bmatrix}$$

$$= \frac{r\omega^2}{n}[-m_1 l_1 \cos 2\alpha - m_2 l_2 \cos 2\beta + m_2 l_2 \cos 2\beta + m_1 l_1 \cos 2\alpha]$$

$$= 0$$

$$\sum C_{sv} = \frac{r\omega^2}{n}\begin{bmatrix} m_1(-l_i)\sin 2\alpha + m_2(-l_2) \\ \sin 2(180° + \beta) + m^2(l_2) \\ \sin 2(180° - \beta) + m_1(l_1) \\ \sin 2(360° - \alpha) \end{bmatrix}$$

$$= \frac{2r\omega^2}{n}[-m_1 l_1 \sin 2\alpha - m_2 l_2 \sin 2\beta]$$

Out of balance secondary couple

$$= \frac{2r\omega^2}{n}[m_1 l_1 \sin 2\alpha + m_2 l_2 \sin 2\beta]$$

Graphical Solution

From the polygon of primary forces (Fig. 14.25),

$$m_1 \cos\alpha = m_2 r \cos\beta$$

or $\frac{m_1}{m_2} = \frac{\cos\beta}{\cos\alpha}$

From the polygon of primary couples,

$$m_2 l_2 \sin\beta = m_1 l_1 \sin\alpha$$

or $\frac{l_1}{l_2} = \frac{m_2 \sin\beta}{m_1 \sin\alpha} = \frac{\cos\alpha\sin\beta}{\cos\beta\sin\alpha} = \frac{\tan\beta}{\tan\alpha}$

From the polygon of secondary forces,

$$m_1 \cos 2\alpha = -m_2 \cos 2\beta$$

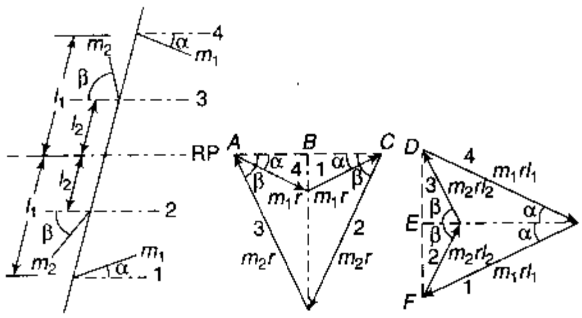
or $m_1 \cos 2\alpha + m_2 \cos 2\beta = 0$

Simplifying as in (iii) of analytical solution above ,

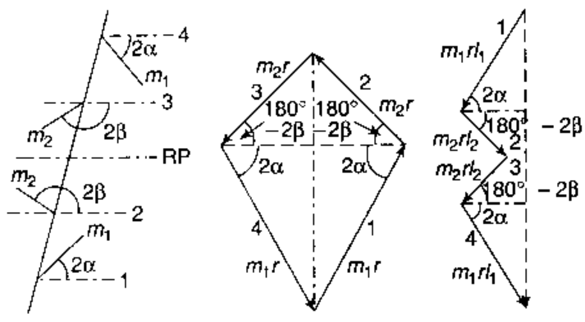
$$\cos\alpha\cos\beta = \frac{1}{2}$$

From the polygon of secondary couples,

Resultant $mrl = m_1 r l_1 \sin 2\alpha + m_2 r l_1 \sin(180° - 2\beta) + m_2 r l_1 \sin(180° - 2\beta) + m_1 r l_1 \sin 2\alpha$



(a) Primary cranks Force polygon Couple polygon



(b) Secondary cranks Force polygon Couple polygon

Fig. 14.25

or out of balance secondary couple

$$= \frac{2r\omega^2}{n}[m_1 l_1 \sin 2\alpha + m_2 l_2 \sin 2\beta]$$

Example 14.14 *Each crank and the connecting rod of a four-crank in-line engine are 200 mm and 800 mm respectively. The outer cranks are set at 120° to each other and each has a reciprocating mass of 200 kg. The spacing between adjacent planes of cranks are 400 mm, 600 mm and 500 mm. If the engine is in complete primary balance, determine the reciprocating masses of the inner cranks and their relative angular positions. Also find the secondary unbalanced force if the engine speed is 210 rpm.*

Solution

$$\omega = \frac{2\pi \times 210}{60} = 22 \text{ rad/s}$$

$n = 800/200 = 4$

Figure 14.26 represents the relative position of the cylinders and the cranks.

Taking 2 as the reference plane,

primary couples about the *RF*,

$m_1 r_1 l_1 = 200 \times 0.2 \times 0.4 = 16$

$m_2 r_2 l_2 = 0$

$m_3 r_3 l_3 = m_3 \times 0.2 \times (-0.6) = -0.12\, m_3$

$m_4 r_4 l_4 = 200 \times 0.2 \times (-1.1) = -44$

The couple polygon is drawn in Fig. 14.26.

$m_3 r_3 l_3$ of the crank 3 from the diagram = 53.7 at 135°

$\therefore m_3 r_3 l_3 = m_3 \times 0.12 = 53.7$ or $m_3 = 448$ kg

As its direction is to be negative, its direction is (135° + 180°) or 315°.

Primary force (mr) along each of outer cranks = $200 \times 0.2 = 40$

Primary force (mr) along crank 3 = $448 \times 0.2 = 89.6$

The force polygon is drawn in Fig. 14.26.

$m_2 r_2$ of crank 2 from the diagram = 87.6 at 161.4°

$\therefore m_2 r_2 = m_2 \times 0.2 = 87.6$ or $m_2 = 438$ kg

Its angular position is 161.4°.

Figure 14.26(b) represents the relative position of the cylinders and the cranks.

From secondary unbalanced force polygon,

$mr = 198$

Maximum unbalanced force

$$= 198 \times \frac{\omega^2}{n} = 198 \times \frac{22^2}{4} = 23\,958 \text{ N}$$

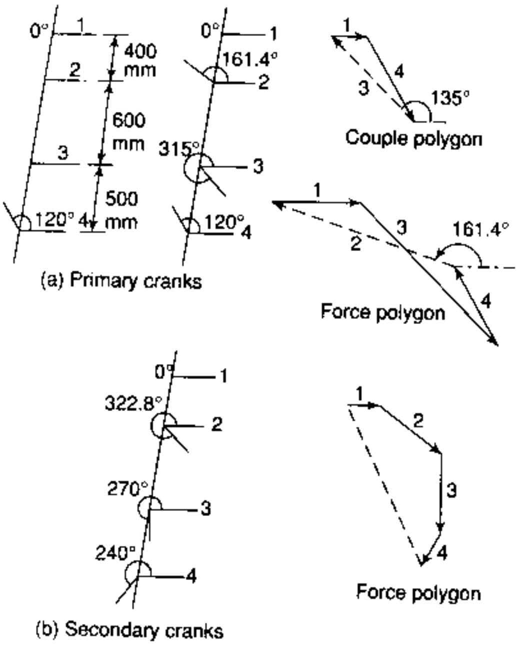


Fig. 14.26

Example 14.15 *The successive cranks of a five-cylinder in-line engine are at 144° apart. The spacing between cylinder centre lines is 400 mm. The lengths of the crank and the connecting rod are 100 mm and 450 mm respectively and the reciprocating mass for each cylinder is 20 kg. The engine speed is 630 rpm. Determine the maximum values of the primary and secondary forces and couples and the position of the central crank at which these occur.*

Solution

$$\omega = \frac{2\pi \times 630}{60} = 66 \text{ rad/s}$$

Figure 14.27(a) represents the relative position of the cylinders and the cranks.

Primary force (mr) along each crank = $20 \times 0.1 = 2$

The primary force polygon is a closed polygon [Fig. 14.27(b)], therefore, no unbalanced primary force.

Primary couples about the mid-plane,

$m_1r_1l_1 = 2 \times 0.8 = 1.6$

$m_2r_2l_2 = 2 \times 0.4 = 0.8$

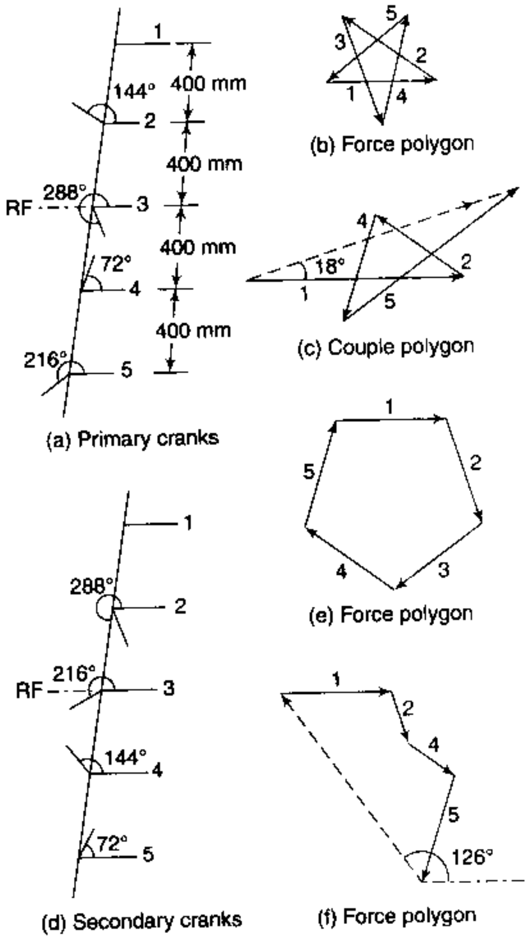


Fig. 14.27

$m_3r_3l_3 = 0$

$m_4r_4l_4 = -0.8$

$m_5r_5l_5 = -1.6$

The couple polygon is drawn in Fig. 14.27(c).

Unbalanced mrl on measurement = 2.1

The unbalanced primary couple = $2.1 \times \omega^2$
$= 2.1 \times 66^2 = 9148$ N

The maximum value of the secondary couple will occur when it coincides with the line of stroke, i.e., when the crankshaft rotates through 18° and 198° clockwise. As initial position of mid-crank 3 is 288°, its positions for maximum primary couple will be (288° - 18°) and (288° - 198°) or 270° and 90°.

The positions of the cranks for secondary forces and couples will as shown in Fig. 14.27(d).

Secondary force (mr) along each crank
$= 20 \times 0.1 = 2$

The force polygon is a closed polygon [Fig. 14.27(e)], therefore, no unbalanced secondary force.

Secondary couples about the mid-plane, $m_1r_1l_1$, $m_2r_2l_2$ ... are the same as above for primary couples.

The couple polygon is shown in Fig. 14.27(f). It does not close.

Unbalanced mrl on measurement = 3.41

The unbalanced couple

$$= 3.41 \times \frac{\omega^2}{n} = 3.41 \times \frac{1}{450/100} \times 66^2$$

$= 3301$ N.m

The maximum value of the secondary couple will occur when it coincide with the line of stroke, i.e., when the crankshaft rotates through 126° and 306° clockwise. As initial position of mid-crank 3 is 216°, its positions for maximum secondary couple will be (216° - 126°), (216° - 306°) or 90° and -90° or 90° and 270°. However, since the secondary crank positions are taken at double the angles, the original crank will rotate through 45° and 135°. As the crank rotates through a full revolution, the maximum secondary couple will also occur at 225° and 315°.

Example 14.16 *Each crank and the connecting rod of a six-cylinder four stroke in-line engine are 60 mm and 240 mm respectively. The pitch distances between the cylinder centre lines are 80 mm, 80 mm, 100 mm, 80 mm and 80 mm respectively. The reciprocating mass of each cylinder is 1.4 kg. The engine speed is 1000 rpm. Determine the out-of-balance primary and secondary forces and couples on the engine if the firing order be 142635. Take a plane midway between the cylinders 3 and 4 as the reference plane.*

Solution Figure 14.28(a) represents the relative position of the cylinders and the cranks for the firing order 142635 for clockwise rotation of the crankshaft. As the engine is a four-stroke engine, firing takes place once in two revolutions of the crank and the angle between the cranks is 120°.

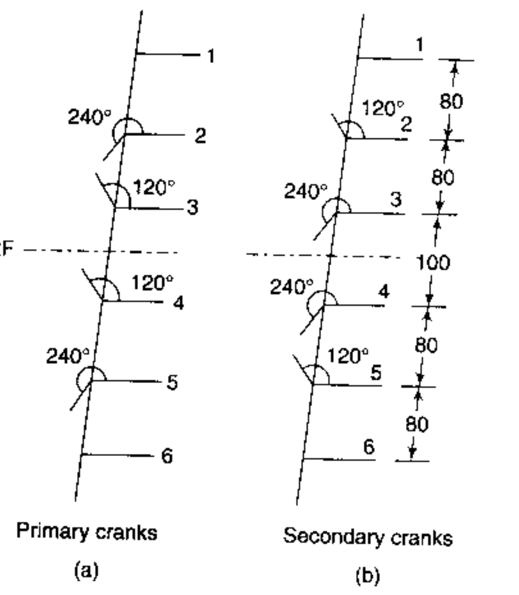


Fig. 14.28

Primary force (mr) along each crank $= 1.4 \times 60 = 84$

The force polygon can exactly be drawn in the same manner as shown in Fig. 14.22(b). It is a closed polygon, therefore, no unbalanced primary force.

Primary couples about the mid-plane,

$$m_1r_1l_1 = 84 \times 210 = 17\,640$$
$$m_2r_2l_2 = 84 \times 130 = 10\,920$$
$$m_3r_3l_3 = 84 \times 50 = 4200$$
$$m_4r_4l_4 = -84 \times 50 = -4200$$
$$m_5r_5l_5 = -84 \times 130 = -10\,920$$
$$m_6r_6l_6 = -84 \times 210 = -17\,640$$

The couple polygon is again exactly similar to as shown in Fig. 14.22(b). It is a closed polygon, therefore, no unbalanced primary couple.

The secondary cranks position is shown in Fig. 14.28(b).

Secondary force (mr) along each crank $= 1.4 \times 60 = 84$

The force polygon can exactly be drawn in the same manner as shown in Fig. 14.22(d). It is a closed polygon, therefore, no unbalanced secondary force.

Secondary couples about the mid-plane,

$m_1r_1l_1$, $m_2r_2l_2$ ...... are the same as above for primary couples.

The couple polygon is again exactly similar to as shown in Fig. 14.22(d). It is a closed polygon, therefore, no unbalanced secondary couple.

Example 14.17 *The stroke of each piston of a six-cylinder two-stroke in-line engine is 320 mm and the connecting rod is 800 mm long. The cylinder centre lines are spaced at 500 mm. The cranks are at 60° apart and the firing order is 145236. The reciprocating mass per cylinder is 100 kg and the rotating parts are 50 kg per crank. Determine the out-of-balance forces and couples about the mid plane if the engine rotates at 200 rpm.*

Solution Figure 14.29(a) represents the relative position of the cylinders and the cranks for the firing order 145236 for clockwise rotation of the crankshaft.

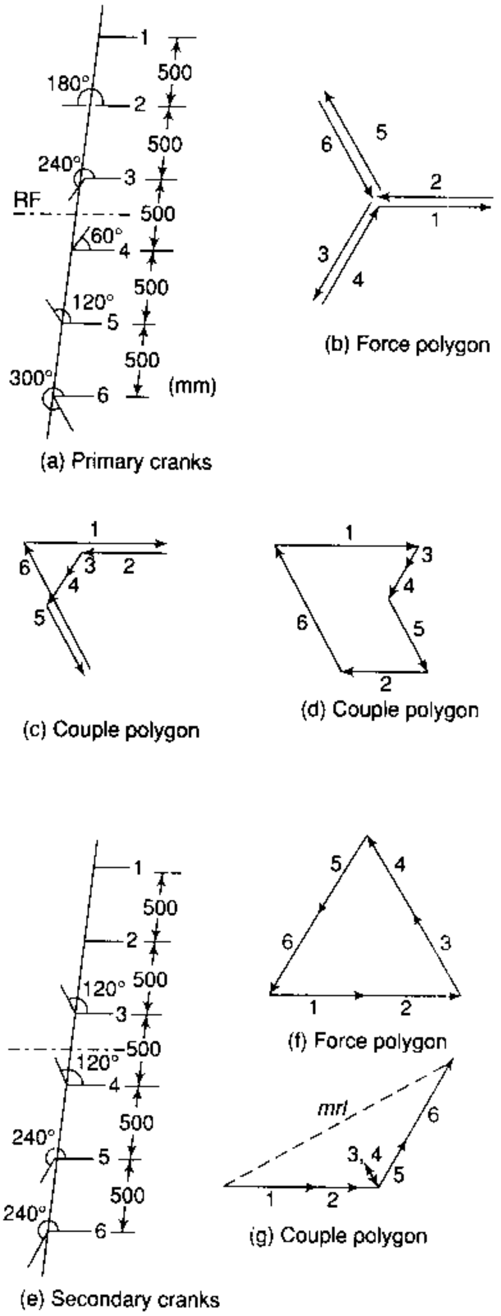


Fig. 14.29

Total mass at the crank pin = 100 + 50 = 150 kg

Primary force (mr) along each crank = 150×0.16 = 24

The force polygon [Fig. 14.29(b)] is a closed polygon, therefore, no unbalanced primary force.

Primary couples about the mid-plane,

$m_1r_1l_1 = 24 \times 1.25 = 30$

$m_2r_2l_2 = 24 \times 0.75 = 18$

$m_3r_3l_3 = 24 \times 0.25 = 6$

$m_4r_4l_4 = -6$

$m_5r_5l_5 = -18$

$m_6r_6l_6 = -30$

The couple polygon [Fig. 14.29(c)] is again a closed polygon, therefore, no unbalanced primary couple. As it is not necessary to add the vectors in order, the couple polygon can also be drawn as in Fig. 14.29(d).

The positions of the cranks for secondary forces and couples will as shown in Fig. 14.29(e). Secondary force (mr) along each crank = 100 × 0.16 = 16

(The rotating masses do not affect the secondary forces as they are only due to second harmonics of the piston acceleration.)

The force polygon is a closed polygon [Fig. 14.29(f)], therefore, no unbalanced secondary force.

Secondary couples about the mid-plane,

$m_1r_1l_1 = 16 \times 1.25 = 20$

$m_2r_2l_2 = 16 \times 0.75 = 12$

$m_3r_3l_3 = 16 \times 0.25 = 4$

$m_4r_4l_4 = -4$

$m_5r_5l_5 = -12$

$m_6r_6l_6 = -20$

The couple polygon is shown in Fig. 14.29(g). It does not close.

Unbalanced mrl on measurement = 55.43.

$$\text{The unbalanced couple} = 55.43 \times \frac{\omega^2}{n}$$

$$= 55.43 \times \frac{1}{5} \times \left(\frac{2\pi \times 200}{60}\right)^2 = 4863 \text{ N.m}$$

Example 14.18 *The cranks of a four-cylinder marine oil engine are arranged at angular intervals of 90°. The engine speed is 70 rpm and the reciprocating mass per cylinder is 800 kg. The inner cranks are 1 m apart and are symmetrically arranged between the outer cranks which are 2.6 m apart. Each crank is 400 mm long.*

Determine the firing order of the cylinders for the best balance of reciprocating masses and also the magnitude of the unbalanced primary couple for that arrangement.

Solution

$m = 800$ kg $\quad N = 70$ rpm

$r = 0.4$ m $\quad \omega = \dfrac{2\pi \times 70}{60} = 7.33$ rad/s

$mr\omega^2 = 800 \times 0.4 \times (7.33)^2 = 17\ 195$

There are four cranks. They can be used in six different arrangements as shown in Fig.14.30(a). It can be observed that in all the cases, primary forces are always balanced. Primary in each case will be as under:

Taking 1 as the reference plane,

$$C_{p1} = mr\omega^2\sqrt{(-l_3)^2 + (l_2 - l_4)^2}$$

$$= 17\ 195\sqrt{(-1.8)^2 + (0.8 - 2.6)^2}$$

$$= 43\ 761 \text{ N.m}$$

$C_{p6} = C_{p1} = 43\ 761$ N.m, only l_2 and l_4 are interchanged.

$$C_{p2} = mr\omega^2\sqrt{(-l_4)^2 + (l_2 - l_3)^2}$$

$$= 17195\sqrt{(-2.6)^2 + (0.8 - 1.8)^2}$$

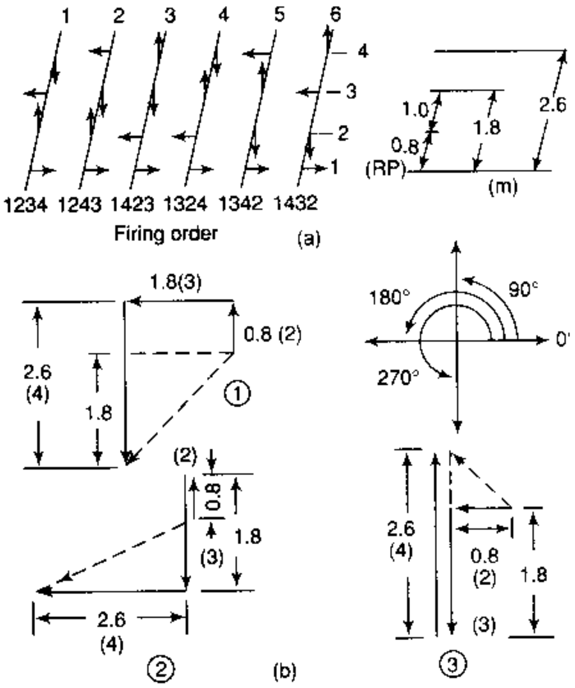


Fig. 14.30

$$= 47\ 905 \text{ N.m}$$

$C_{p5} = C_{p2} = 47\ 905$ N.m, l_2 and l_3 are interchanged.

$$C_{p3} = mr\omega^3\sqrt{(-l_2)^2 + (l_4 - l_3)^2}$$

$$= 17195\sqrt{(-0.8)^2 + (2.6 - 1.8)^2}$$

$$= 19\ 448 \text{ N.m}$$

$C_{p4} = C_{p3} = 19\ 448$ N.m, l_4 and l_3 are interchanged.

Thus the best arrangement is of 3rd and 4th. The firing orders are 1423 and 1324 respectively.

Unbalanced couple = 19448 N.m

Graphical solution has also been shown in Fig. 14.30(b).

Example 14.19 *The intermediate cranks of a four-cylinder symmetrical engine, which is in complete primary balance, are at 90° to each other and each has a reciprocating mass of 400 kg. The centre distance between intermediate cranks is 600 mm and between extreme cranks, it is 1800 mm. Lengths of the connecting rods and the cranks are 900 mm and 200 mm respectively. Calculate the masses fixed to the extreme cranks with their relative angular positions. Also, find the magnitude of the secondary forces and couples about the centre line of the system if the engine speed is 500 rpm.*

Solution Refer Fig.14.31.

$l = 0.9$ m $\quad m_2 = m_3 = 400$ kg

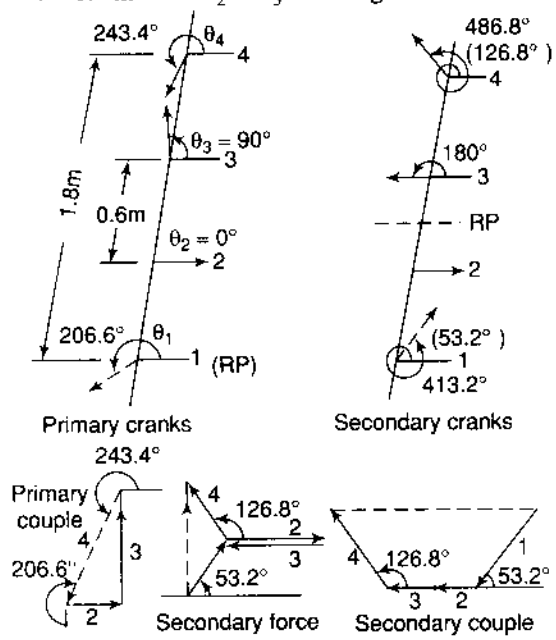


Fig. 14.31

$r = 0.2$ m $\quad n = \dfrac{0.9}{0.2} = 4.5$

The engine is in complete primary balance. Taking 1 as the reference plane,

$m_2 r_2 l_2 = 400 \times 0.2 \times 0.6 = 48$

$m_3 r_3 l_3 = 400 \times 0.2 \times 1.2 = 96$

$m_4 r_4 l_4 = m_4 \times 0.2 \times 1.8 = 0.36\, m_4$

$$0.36 m_4 = \sqrt{\begin{matrix}(48\cos 0° + 96\cos 90°)^2 \\ +(48\sin 0° + 96\sin 90°)^2\end{matrix}}$$

$$= \sqrt{(48)^2 + (96)^2}$$

$= 107.33$

$m_4 = \underline{298 \text{ kg}}$

$\tan\theta_4 = \dfrac{-96}{-48} = 2; \theta_4 = \underline{243.4°}$

By symmetry, $m_1 = \underline{298 \text{ kg}}$

and $\tan\theta_1 = \dfrac{-48}{-96} = 0.5; \theta_1 = 206.6°$

The position of the cranks for secondary forces and couples will be such that the angles are doubled (Fig.14.31).

$$\omega = \frac{2\pi \times 150}{60} = 15.7 \text{ rad/s}$$

Secondary force

$$= \frac{r\omega^2}{n}\begin{bmatrix}\{298(\cos 53.2° + \cos 126.8°) + 400 \\ (\cos 0° + \cos 180°)\}^2 \\ +\{298(\sin 53.2° + \sin 126.8°) + 400 \\ (\sin 0° + \sin 180°)\}^2\end{bmatrix}^{1/2}$$

$$= \frac{0.2 \times (15.7)^2}{4.5}(\sin 53.2° + \sin 126.8°) \times 298$$

$= \underline{5233.6 \text{ N}}$

Secondary couple about the centre line

$$= \frac{r\omega^2}{n}\begin{bmatrix}\{298(-0.9\cos 53.2° + 0.9\cos 126.8°) \\ +400(-0.3\cos 0° + 0.3\cos 180°)^2\} \\ +\{298(-0.9\sin 53.2° + 0.9\sin 126.8°) \\ +400(-0.3\sin 0° + 0.3\sin 180°)\}^2\end{bmatrix}^{1/2}$$

$$= \frac{0.2 \times (15.7)^2}{4.5}[298 \times (-0.9 \cos 53.2° + 0.9 \cos 126.8°) \times 400 \times (-0.6)]$$

$= \underline{6155 \text{ N.m}}$

14.11 BALANCING OF V-ENGINES

In V-engines, a common crank OA is operated by two connecting rods OB_1 and OB_2. Figure 14.32 shows a symmetrical two cylinder V-cylinder, the centre lines of which are inclined at an angle α to the x-axis.

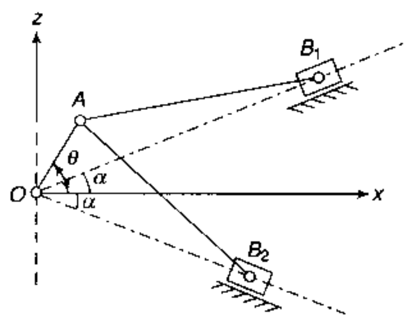


Fig. 14.32

Let θ be the angle moved by the crank from the x-axis.

Primary force

Primary force of 1 along line of stroke $OB_1 = mr\omega^2 \cos(\theta - \alpha)$

Primary force of 1 along x-axis $= mr\omega^2 \cos(\theta - \alpha) \cos\alpha$

Primary force of 2 along line of stroke $OB_2 = mr\omega^2 \cos(\theta + \alpha)$

Primary force of 2 along the x-axis $= mr\omega^2 \cos(\theta + \alpha) \cos\alpha$

Total primary force along x-axis

$= mr\omega^2 \cos\alpha[\cos(\theta - \alpha) + \cos(\theta + \alpha)]$

$= mr\omega^2 \cos\alpha\,[(\cos\theta\cos\alpha + \sin\theta\sin\alpha) + (\cos\theta\cos\alpha - \sin\theta\sin\alpha)]$

$= mr\omega^2 \cos\alpha\, 2\cos\theta\cos\alpha$

$= 2mr\omega^2 \cos^2\alpha \cos\theta \quad (14.29)$

Similarly, total primary force along the z-axis

$mr\omega^2 [\cos(\theta - \alpha)\sin\alpha - \cos(\theta + \alpha)\sin\alpha]$

$$= mr\omega^2 \sin\alpha\,[(\cos\theta\cos\alpha + \sin\theta\sin\alpha) - (\cos\theta\cos\alpha - \sin\theta\sin\alpha)]$$

$$= mr\omega^2 \sin\alpha\, 2\sin\theta\sin\alpha$$

$$= 2mr\omega^2 \sin^2\alpha\sin\theta \qquad (14.30)$$

Resultant primary force

$$= \sqrt{(2mr\omega^2\cos^2\alpha\cos\theta)^2 + (2mr\omega^2\sin^2\alpha\sin\theta)^2}$$

$$= 2mr\omega^2\sqrt{(\cos^2\alpha\cos\theta)^2 + (\sin^2\alpha\sin\theta)^2} \qquad (14.31)$$

It will be at an angle β with the x-axis, given by

$$\tan\beta = \frac{\sin^2\alpha\sin\theta}{\cos^2\alpha\cos\theta} \qquad (14.32)$$

If $2\alpha = 90°$, resultant force

$$= 2mr\omega^2\sqrt{(\cos^2 45°\cos\theta)^2 + (\sin^2 45°\sin\theta)^2}$$

$$= mr\omega^2 \qquad (14.33)$$

$$\tan\beta = \frac{\sin^2 45°\sin\theta}{\cos^2 45°\cos\theta} = \tan\theta \qquad (14.34)$$

i.e., $\beta = \theta$ or it acts along the crank and, therefore, can be completely balanced by a mass at a suitable radius diametrically opposite to the crank such that $m_r r_r = mr$.

For a given value of α, the resultant primary force is maximum when

$(\cos^2\alpha\cos\theta)^2 + (\sin^2\alpha\sin\theta)^2$ is maximum

or $(\cos^4\alpha\cos^2\theta + \sin^4\alpha\sin^2\theta)$ is maximum

or $\frac{d}{d\theta}(\cos^4\alpha\cos^2\theta + \sin^4\alpha\sin^2\theta) = 0$

or $-\cos^4\alpha.2\cos\theta\sin\theta + \sin^4\alpha.2\sin\theta\cos\theta = 0$

or $-\cos^4\alpha.\sin 2\theta + \sin^4\alpha.\sin 2\theta = 0$

or $\sin 2\theta(\sin^4\alpha - \cos^4\alpha) = 0 \qquad (14.35)$

As α is not zero, therefore, for a given value of α, the resultant primary force is maximum when θ is zero degree.

Secondary force

Secondary force of 1 along $OB_1 = \frac{mr\omega^2}{n}\cos 2(\theta - \alpha)$

Secondary force of 1 along x-axis $= \frac{mr\omega^2}{n}\cos 2(\theta - \alpha)\cos\alpha$

Secondary force of 2 along $OB_2 = \frac{mr\omega^2}{n}\cos 2(\theta + \alpha)$

Secondary force of 2 along x-axis $= \frac{mr\omega^2}{n}\cos 2(\theta + \alpha)\cos\alpha$

Total secondary force along x-axis

$$= \frac{mr\omega^2}{n} \cos\alpha[\cos 2(\theta - \alpha) + \cos 2(\theta + \alpha)]$$

$$= \frac{mr\omega^2}{n} \cos\alpha[(\cos 2\theta \cos 2\alpha + \sin 2\theta \sin 2\alpha) + (\cos 2\theta \cos 2\alpha - \sin 2\theta \sin 2\alpha)]$$

$$= \frac{2mr\omega^2}{n} \cos\alpha \cos 2\theta \cos 2\alpha \quad (14.36)$$

Similarly, secondary force along z-axis $= \dfrac{2mr\omega^2}{n} \sin\alpha \sin 2\theta \sin 2\alpha$ (14.37)

Resultant secondary force

$$= \frac{2mr\omega^2}{n} \sqrt{(\cos\alpha \cos 2\theta \cos 2\alpha)^2 + (\sin\alpha \sin 2\theta \sin 2\alpha)^2} \quad (14.38)$$

$$\tan\beta' = \frac{\sin\alpha \sin 2\theta \sin 2\alpha}{\cos\alpha \cos 2\theta \cos 2\alpha} \quad (14.39)$$

If $2\alpha = 90°$ or $\alpha = 45°$,

$$\text{Secondary force} = \frac{2mr\omega^2}{n} \sqrt{\left(\frac{\sin 2\theta}{\sqrt{2}}\right)^2}$$

$$= \sqrt{2} \frac{mr\omega^2}{n} \sin 2\theta \quad (14.40)$$

$$\tan\beta' = \infty, \beta' = 90° \quad (14.41)$$

This means that the force acts along z-axis and is a harmonic force and special methods are needed to balance it.

Example 14.20 *The cylinder axes of a V-engine are at right angles to each other. The weight of each piston is 2 kg and of each connecting rod is 2.8 kg. The weight of the rotating parts like crank webs and the crank pin is 1.8 kg. The connecting rod is 400 mm long and its centre of mass is 100 mm from the crank-pin centre. The stroke of the piston is 160 mm. Show that the engine can be balanced for the revolving and the primary force by a revolving countermass. Also, find the magnitude and the position if its centre of mass from the crankshaft centre is 100 mm.*

What is the value of the resultant secondary force if the speed is 840 rpm?

Solution

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 840}{60} = 88 \text{ rad/s}$$

$$n = \frac{400}{80} = 5$$

Total mass of rotating parts at the crank pin

$$= 1.8 + \frac{2.8 \times (400 - 100)}{400} \times 2$$

$= 6$ kg

Unbalanced force due to revolving mass along the crank $= 6\ r\omega^2$

Total mass of reciprocating parts/cylinder

$$= 2 + \frac{2.8 \times 100}{400}$$

$= 2.7$ kg

As the angle between the cranks is 90°, i.e., $2\alpha = 90°$,

$\therefore$ The resultant primary force $= mr\omega^2 = 2.7\ r\omega^2$ (Eq. 14.33)

It acts along the crank. (Eq. 14.34)

Total unbalanced force along the crank

$= (6 + 2.7)\ r\omega^2 = 8.7\ r\omega^2$

It can easily be balanced by a revolving mass in a direction opposite to that of crank.

Countermass m_r at a radial distance of 100 mm,

$m_r \times 100 \times \omega^2 = 8.7 \times (160/2)\omega^2$

$m_r = 6.96$ kg

The rotating masses do not affect the secondary forces as they are only due to second harmonics of the piston acceleration.

Secondary force $= \sqrt{2}\,\dfrac{mr\omega^2}{n} \sin 2\theta$ (Eq. 14.40)

$= \sqrt{2} \times \dfrac{2.7 \times 0.08 \times 88^2}{5} \sin 2$

$= 473.1 \sin 2\theta$

Maximum value at $\theta = 45° = 473.1$ N

Example 14.21 *The cylinders of a twin V-engine are set at 60° angle with both pistons connected to a single crank through their respective connecting rods. Each connecting rod is 600 mm long and the crank radius is 120 mm. The total rotating mass is equivalent to 2 kg at the crank radius and the reciprocating mass is 1.2 kg per piston. A balance mass is also fitted opposite to the crank equivalent to 2.2 kg at a radius of 150 mm. Determine the maximum and minimum values of the primary and secondary forces due to inertia of the reciprocating and the rotating masses if the engine speed is 800 rpm.*

Solution Refer Fig. 14.33.

$m = 1.2$ kg $\qquad M = 2$ kg

$l = 600$ mm $\qquad r = 120$ mm

$m' = 2.2$ kg $\qquad r' = 150$ mm

$N = 800$ rpm

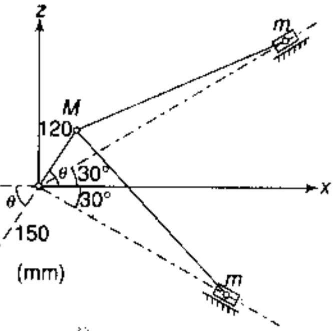


Fig. 14.33

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1050}{60} = 110 \text{ rad/s}$$

$$n = \frac{400}{80} = 5$$

Primary force

Total primary force along x-axis

$= 2mr\omega^2 \cos^2 \alpha \cos \theta$ (Eq. 14.29)

Centrifugal force due to rotating mass along x-axis $= Mr\omega^2 \cos\theta$

Centrifugal force due to balancing mass along x-axis $= -m' r\omega^2 \cos\theta$

Total unbalanced force along x-axis

$= 2mr\omega^2 \cos^2 \alpha \cos \theta + Mr\omega^2 \cos \theta - m'r'\omega^2 \cos \theta$

$= \omega^2 \cos \theta (2mr \cos^2 \alpha + Mr - m'r')$

$= 110^2 \times \cos \theta (2 \times 1.2 \times 0.12 \cos^2 30° + 2 \times 0.12 - 2.2 \times 0.15)$

$= 110^2 \times \cos \theta (0.216 + 0.24 - 0.33)$

$= 1524.6 \cos \theta$ N

Total primary force along z-axis

$= 2mr\omega^2 \sin^2 \alpha \sin \theta$ (Eq. 14.30)

Centrifugal force due to rotating mass along z-axis $= Mr\omega^2 \sin\theta$

Centrifugal force due to balancing mass along z-axis $= -m'r\omega^2 \sin\theta$

Total unbalanced force along z-axis

$= 2mr\omega^2 \sin^2 \alpha \sin \theta + Mr\omega^2 \sin\theta - m'r'\omega^2 \sin\theta$

$= \omega^2 \sin\theta (2mr \sin^2 \alpha + Mr - m'r')$

$= 110^2 \times \sin \theta (2 \times 1.2 \times 0.12 \sin^2 30° + 2 \times 0.12 - 2.2 \times 0.15)$

$= 110^2 \times \sin \theta (0.072 + 0.24 - 0.33)$

$= -217.8 \sin \theta$ N

Resultant primary force

$= \sqrt{1524^2 \cos^2 \theta + (-217.8)^2 \sin^2 \theta}$

$= \sqrt{2\ 322\ 576 \cos^2 \theta + 47\ 437 \sin^2 \theta}$

$= \sqrt{\dfrac{2\ 275\ 139 \cos^2 \theta + 47\ 437}{\cos^2 \theta + 47\ 437 \sin^2 \theta}}$

$= \sqrt{2\ 275\ 139 \cos^2 \theta + 47\ 437}$

This is maximum when θ is 0° and minimum when $\theta = 90°$.

Maximum primary force

$= \sqrt{2\ 275\ 139 + 47\ 437} = 1524$ N

Minimum primary force

$= \sqrt{47\ 437} = 217.8$ N

Secondary force

The rotating masses do not affect the secondary forces as they are only due to second harmonics of the piston acceleration.

Resultant secondary force

$$= \frac{2mr\omega^2}{n}\sqrt{\begin{matrix}(\cos\alpha\cos 2\theta\cos 2\alpha)^2 \\ +(\sin\alpha\sin 2\theta\sin 2\alpha)^2\end{matrix}} \qquad \text{(Eq. 14.38)}$$

$$= \frac{2\times 1.2\times 0.12\times 110^2}{5}\sqrt{\begin{matrix}(\cos 30^\circ\cos 2\theta\cos 60^\circ)^2 \\ +(\sin 30^\circ\sin 2\theta\sin 60^\circ)^2\end{matrix}}$$

$$= 696.96\sqrt{(0.433\cos 2\theta)^2 + (0.433\sin 2\theta)^2}$$

This is maximum when θ is 0° and minimum when $\theta = 90°$

Maximum primary force

$= 696.96 \times 0.433 = 301.8$ N

Minimum primary force

$= 696.96 \times 0.433 = 301.8$ N

Thus, the secondary force has the same value for maximum and minimum.

14.12 BALANCING OF W, V-8 AND V-12 ENGINES

In W-engines, a common crank OA is operated by three connecting rods as shown in Fig. 14.34.

Primary force

Primary force of 1 along x-axis$= mr\omega^2\cos(\theta - \alpha)\cos\alpha$

Primary force of 2 along x-axis $= mr\omega^2\cos(\theta + \alpha)\cos\alpha$

Primary force of 3 along x-axis $= mr\omega^2\cos\theta\cos\alpha$

$= mr\omega^2\cos\theta$ (as $\alpha = 0°$)

Total primary force along x-axis

$= mr\omega^2\cos\alpha[\cos(\theta - \alpha) + \cos(\theta + \alpha)] + mr\omega^2\cos\theta$

$= 2\,mr\omega^2\cos^2\alpha\cos\theta\cos\alpha + mr\omega^2\cos\theta$

$= mr\omega^2\cos\theta\,(2\cos^2\alpha + 1)$

Total primary force along z-axis will be same as in the V-twin engine because

Primary force of 3 along z-axis $= mr\omega^2\cos\theta\sin\alpha$

$= 0$

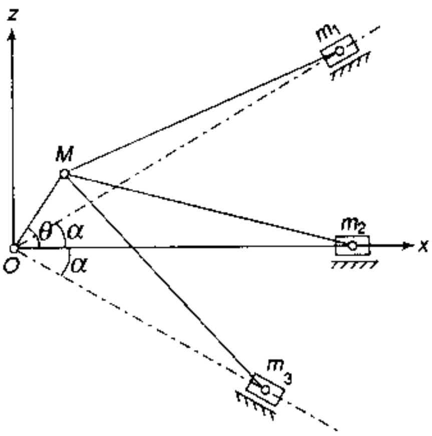


Fig. 14.34

Resultant primary force

$$= \sqrt{[mr\omega^2\cos\theta(2\cos^2\alpha + 1)^2] + (2mr\omega^2\sin^2\alpha\sin\theta)^2}$$

$$= mr\omega^2\sqrt{[\cos\theta(2\cos^2\alpha + 1)^2] + (2\sin^2\alpha\sin\theta)^2}$$

It will be at an angle β with the x-axis, given by

$$\tan\beta = \frac{2\sin^2\alpha\sin\theta}{\cos\theta(2\cos^2\alpha + 1)}$$

If $\alpha = 60°$, resultant force

$$= mr\omega^2\sqrt{[\cos\theta(2\cos^2 60° + 1)^2] + (2\sin^2 60°\sin\theta)^2}$$

$$= \frac{3}{2}mr\omega^2$$

$$\tan\beta = \frac{2\sin^2\alpha\sin\theta}{\cos\theta(2\cos^2\alpha + 1)}$$

$$= \frac{2\sin^2 60^\circ \alpha \sin\theta}{\cos\theta(2\cos^2 60^\circ + 1)}$$

$$= \tan\theta$$

i.e., $\beta = \theta$ or it acts along the crank and, therefore, can be completely balanced by a mass at a suitable radius diametrically opposite to the crank such that $m_r r_r = mr$.

Secondary force

Total secondary force along x-axis

$$= \frac{mr\omega^2}{n}\cos\alpha[\cos 2(\theta-\alpha) + \cos 2(\theta+\alpha)] + \frac{mr\omega^2}{n}\cos 2\theta$$

$$= \cos 2\theta\left(\frac{2mr\omega^2}{n}\cos\alpha\cos 2\alpha + 1\right)$$

Total primary force along z-axis will be same as in the V-twin engine.

Resultant secondary force

$$= \frac{mr\omega^2}{n}\sqrt{[\cos 2\theta(2\cos\alpha\cos 2\alpha + 1)]^2 + (2\sin\alpha\sin 2\theta\sin 2\alpha)^2}$$

$$\tan\beta' = \frac{2\sin\alpha\sin 2\theta\sin 2\alpha}{\cos 2\theta(2\cos\alpha\cos 2\alpha + 1)}$$

If $\alpha = 60°$,

secondary force along x-axis $= \frac{mr\omega^2}{2n}\cos 2\theta$

secondary force along z-axis $= \frac{3mr\omega^2}{2n}\sin 2\theta$

It is not possible to balance these forces simultaneously.

V-8 Engine

A V-8 engine consists of two banks of four cylinders each. The two banks are inclined to each other in the shape of a V. The analysis of such an engine will depend upon the arrangement of cylinders in each bank.

Let the cranks of four cylinders on one bank be arranged as shown in Fig. 14.19. In this case there is only a secondary unbalance force equal to $= \frac{4mr\omega^2}{n}$

If the angle between the two banks is 90°,

secondary force $= \sqrt{2}\,\frac{4mr\omega^2}{n}\sin 2\theta$ along z-axis (Eqs 14.40 and 14.41)

V-12 Engine

A V-12 engine consists of two banks of six cylinders each. The two banks are inclined to each other in the shape of V and the analysis depends upon the arrangement of cylinders in each bank.

Let the cranks of six cylinders on one bank are arranged as shown in Fig. 14.22. In this case there is no unbalanced force or couple and thus the engine is completely balanced.

Arrangement of cranks of a V-12 engine

14.13 BALANCING OF RADIAL ENGINES

A radial engine is a multicylinder engine in which all the connecting rods are connected to a common crank. The analysis of forces in such type of engines is much simplified by using the method of *direct and reverse cranks*. As all the forces are in the same plane, no unbalance couples exist.

In a reciprocating engine [Fig. 14.35(a)],

Primary force = $mr\omega^2 \cos\theta$ (along line of stroke)

In the method of direct and reverse cranks, a force identical to this force is generated by two masses in the following way:

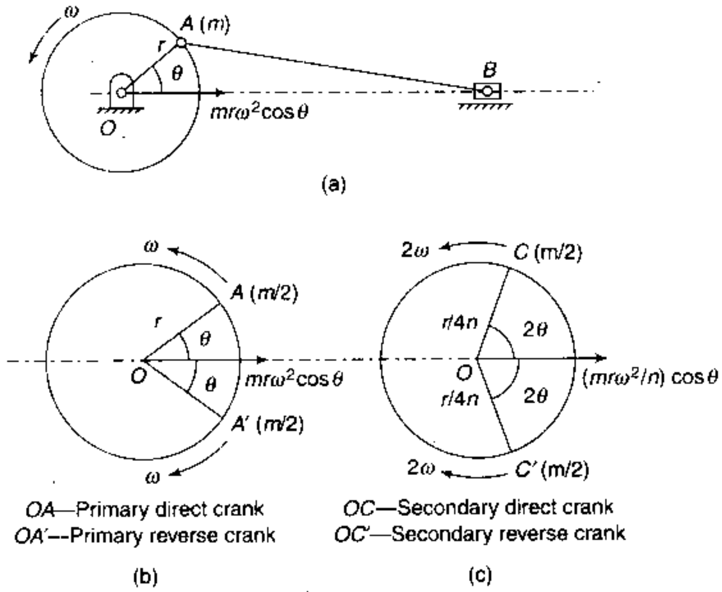


Fig. 14.35

- A mass $m/2$, placed at the crank pin A and rotating at an angular velocity ω in the given direction [Fig. 14.35(b)].
- A mass $m/2$, placed at the crank pin of an imaginary crank OA' at the same angular position as the real crank but in the opposite direction of the line of stroke. This imaginary crank is assumed to rotate at the same angular velocity ω in the opposite direction to that of the real crank. Thus, while rotating; the two masses coincide only on the cylinder centre line. Now, the components of centrifugal force due to rotating masses along line of stroke are

  Due to mass at $A = \frac{m}{2} r\omega^2 \cos\theta$

  Due to mass at $A' = \frac{m}{2} r\omega^2 \cos\theta$

Thus, total force along line of stroke = $mr\omega^2 \cos\theta$ which is equal to the primary force. At any instant, the components of the centrifugal forces of these two masses normal to the line of stroke will be equal and opposite.

The crank rotating in the direction of engine rotation is known as the *direct crank* and the imaginary crank rotating in the opposite direction is known as the *reverse crank*.

Now,

$$\text{Secondary accelerating force} = mr\omega^2 \frac{\cos 2\theta}{n}$$

$$= mr(2\omega)^2 \frac{\cos 2\theta}{4n}$$

$$= m\frac{r}{4n}(2\omega)^2\cos 2\theta \qquad \text{(along line of stroke)}$$

This force can also be generated by two masses in a similar way as follows:

- A mass $m/2$, placed at the end of direct secondary crank of length $r/(4n)$ at angle 2θ and rotating at an angular velocity 2ω in the given direction [Fig. 14.35(c)].
- A mass $m/2$, placed at the end of reverse secondary crank of length $r/(4n)$ at angle -2θ rotating at an angular velocity 2ω in the opposite direction. Now, the components of centrifugal force due to rotating masses along line of stroke are

Due to mass at $C = \frac{m}{2}\frac{r}{4n}(2\omega)^2\cos 2\theta = \frac{mr\omega^2}{2n}\cos 2\theta$

Due to mass at $C' = \frac{m}{2}\frac{r}{4n}(2\omega)^2\cos 2\theta = \frac{mr\omega^2}{2n}\cos 2\theta$

Thus total force along line of stroke $= 2\times\frac{m}{2}\frac{r}{4n}(2\omega)^2\cos 2\theta = \frac{mr\omega^2}{n}\cos 2\theta$

which is equal to the secondary force.

This method can also be used to find the forces in V-engines.

Example 14.22 *The axes of a three-cylinder air compressor are at 120° to one another and their connecting rods are coupled to a single crank. The length of each connecting rod is 240 mm and the stroke is 160 mm. The reciprocating parts have a mass of 2.4 kg per cylinder. Determine the primary and secondary forces if the engine runs at 2000 rpm.*

Solution

$r = 0.160/2 = 0.08$ m $\qquad l = 0.24$ m

$N = 2000$ rpm $\qquad m = 2.4$ kg

$n = l/r = 0.24/0.08 = 3$

The position of three cylinders is shown in Fig. 14.36.

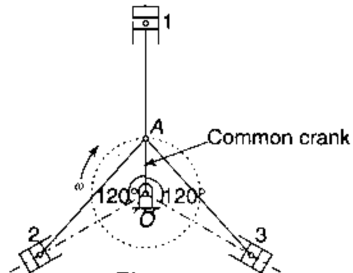


Fig. 14.36

Primary cranks

The primary direct and reverse crank positions are shown in Fig. 14.37 (a) and (b) respectively.

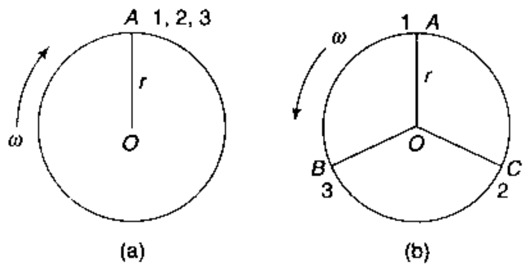


Fig. 14.37

For cylinder 1 From the line of stroke as $\theta = 0°$, the direct and the reverse cranks coincide with the common crank, i.e., along OA.

For cylinder 2 From the line of stroke as $\theta = 120°$, the direct crank is 120° clockwise (along OA) and the reverse crank 120° counter-clockwise (along OC).

For cylinder 3 From the line of stroke as $\theta = 240°$, the direct crank is 240° clockwise (along OA) and the reverse crank 240° counter-clockwise (along OB).

In positions of the direct and reverse cranks are shown in Table 14.1.

Table 14.1

Cylinder	*Crank angle (counter-clockwise)*	*Angle of rotation of the crank*	*Position of direct crank on clockwise rotation*	*Position of reverse crank on counter-clockwise rotation*
1	0°	0°	*OA*	*OA*
2	120°	120°	*OA*	*OC*
3	240°	240°	*OA*	*OB*

Figure 14.37 (b) indicates that the primary reverse cranks form a balanced system and therefore, unbalanced primary force is due to direct cranks only and is given by

Maximum primary force = $3\,\frac{m}{2}\,r\omega^2$

$$= 3 \times \frac{2.4}{2} \times 0.08 \times \left(\frac{2\pi \times 2000}{60}\right)^2$$

$= 3 \times 1.2 \times 0.08 \times 43\,865$

$= 12\,633$ N or <u>12.633 kN</u>

Secondary Cranks

The secondary direct and reverse crank positions are shown in Fig. 14.38(a) and (b) respectively. Refer Table 14.2.

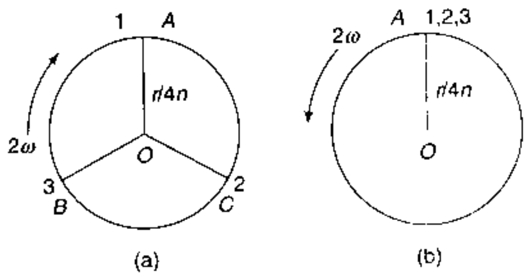


Fig. 14.38

Figure 14.38(a) indicates that the primary direct cranks form a balanced system and therefore, unbalanced secondary force is due to reverse only cranks and is given by

Maximum secondary force

$$= 3\frac{mr\omega^2}{2n} = 3 \times \frac{2.4 \times 0.08}{2 \times 3} \times \left(\frac{2\pi \times 2000}{60}\right)^2$$

$= 3 \times 0.032 \times 43\,865$

$= 4211$ N or <u>4.211 kN</u>

Example 14.23 *The length of each connecting rod of a 60° V-engine is 220 mm and the stroke is 100 mm. The mass of the reciprocating parts is 1.2 kg per cylinder and the crank speed is 2400 rpm. Find the values of the primary and the secondary forces.*

Solution

$r = 0.1/2 = 0.05$ m $\qquad l = 0.22$ m

$N = 2400$ rpm $\qquad m = 1.2$ kg

$n = l/r = 0.22/0.05 = 4.4$

The position of the two cylinders is shown in Fig. 14.39.

Table 14.2

Cylinder	*Crank angle (counter-clockwise)*	*Angle of rotation of the crank*	*Position of direct crank on clockwise rotation*	*Position of reverse crank on counter-clockwise rotation*
1	0°	0°	*OA*	*OA*
2	120°	240°	*OC*	*OA*
3	240°	480° or 120°	*OB*	*OA*

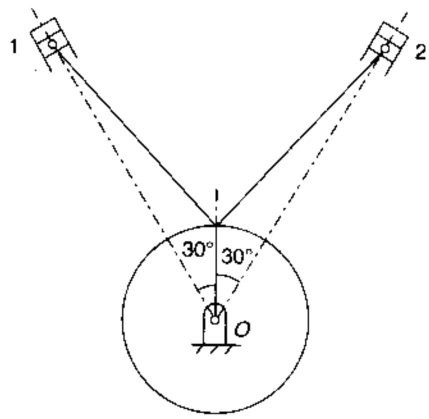


Fig. 14.39

Primary Cranks

The primary direct and reverse crank positions are shown in Fig. 14.40 (a) and (b) respectively. Refer Table 14.3.

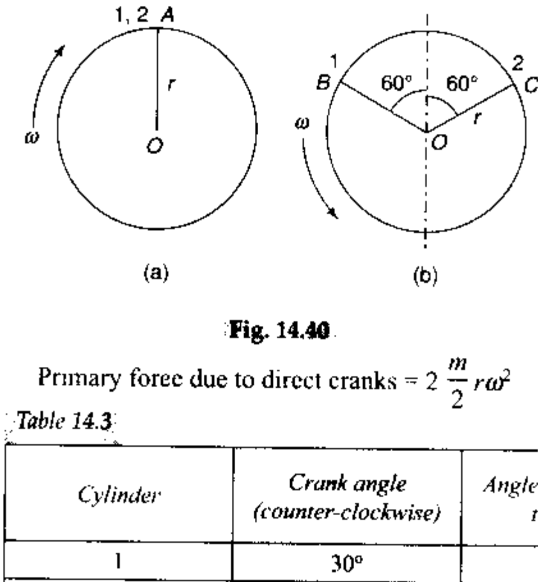


Fig. 14.40

Primary force due to direct cranks $= 2\,\dfrac{m}{2}\,r\omega^2$

$$= 2\,\frac{1.2}{2} \times 0.05 \times \left(\frac{2\pi \times 2400}{60}\right)^2$$

$$= 3 \times 0.6 \times 0.05 \times 63\,165$$

$$= 3790 \text{ N}$$

Primary force due to reverse cranks $= 2\,\dfrac{m}{2}\,r\omega^2 \cos 60^\circ = 3790 \times 0.5 = 1895$ N

Total primary force = 3790 + 1895 = 5685 N

Secondary Cranks

The secondary direct and reverse crank positions are shown in Fig. 14.41(a) and (b) respectively. Refer Table 14.4.

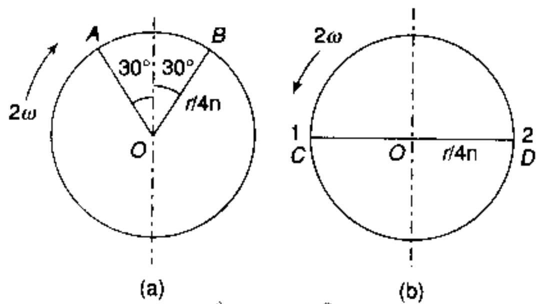


Fig. 14.41

Figure 14.41(b) indicates that the secondary reverse cranks form a balanced system and therefore, unbalanced secondary force is due to direct cranks only and is given by

Thus unbalanced secondary force

$$= 2\,\frac{mr\omega^2}{2n} \cos 30^\circ = 2\,\frac{mr\omega^2}{2}\,\frac{\cos 30^\circ}{n}$$

$$= 3790 \times \frac{\cos 30^\circ}{4.4}$$

$$= 746 \text{ N}$$

Table 14.3

Cylinder	*Crank angle (counter-clockwise)*	*Angle of rotation of the crank*	*Position of direct crank on clockwise rotation*	*Position of reverse crank on counter-clockwise rotation*
1	30°	30°	*OA*	*OB*
2	330°	330°	*OA*	*OC*

Table 14.4

Cylinder	*Crank angle (counter-clockwise)*	*Angle of rotation of the crank*	*Position of direct crank on clockwise rotation*	*Position of reverse crank on counter-clockwise rotation*
1	30°	60°	*OB*	*OC*
2	330°	660° or 300°	*OA*	*OD*

Example 14.24 ***A radial aero-engine has seven cylinders equally spaced with all the connecting rods coupled to a common crank. The crank and each of the connecting rods are 200 mm and 800 mm respectively. The reciprocating mass per cylinder is 3 kg. Determine the magnitude and the angular position of the balance masses required at the crank radius for complete primary and secondary balancing of the engine.***

Solution The position of the seven cylinders is shown in Fig. 14.42.

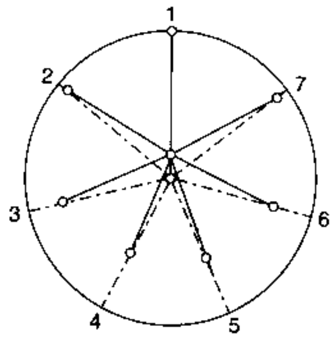


Fig. 14.42

Primary Cranks

The primary direct and reverse crank positions are shown in Fig. 14.43(a) and (b) respectively. Refer Table 14.5.

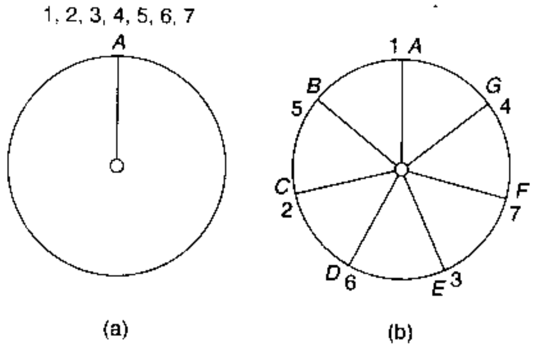


Fig. 14.43

Let 360°/7 = X

This shows that there is primary unbalance due to direct cranks.

Secondary Cranks

The secondary direct and reverse crank positions are shown in Fig. 14.44 (a) and (b) respectively. Refer Table 14.6.

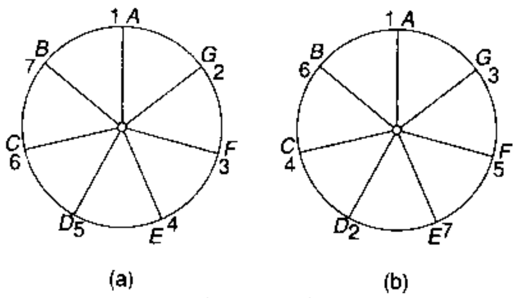


Fig. 14.44

Table 14.5

Cylinder	***Crank angle (counter-clockwise) deg.***	*Angle of rotation of the crank, deg.*	*Position of direct crank on clockwise rotation*	*Position of reverse crank on counter-clockwise rotation*
1	**0**	0	*OA*	*OA*
2	**X**	X	*OA*	*OC*
3	**$2X$**	$2X$	*OA*	*OE*
4	**$3X$**	$3X$	*OA*	*OG*
5	**$4X$**	$4X$	*OA*	*OB*
6	**$5X$**	$5X$	*OA*	*OD*
7	**$6X$**	$6X$	*OA*	*OF*

Table 14.6

Cylinder	*Crank angle (counter-clockwise)*	*Angle of rotation of the crank*	*Position of direct crank on clockwise rotation*	*Position of reverse crank on counter-clockwise rotation*
1	0	0	*OA*	*OA*
2	*X*	2*X*	*OG*	*OD*
3	2*X*	4*X*	*OF*	*OG*
4	3*X*	6*X*	*OE*	*OC*
5	4*X*	8*X* or *X*	*OD*	*OF*
6	5*X*	10*X* or 3*X*	*OC*	*OB*
7	6*X*	12*X* or 5*X*	*OB*	*OE*

There is no unbalance due to secondary direct or inverse cranks.

The unbalanced primary force along the crank can be balanced by a countermass at the crank radius opposite to the crank at 180°.

$$m_c r\omega^2 = \frac{1}{2} \times 7mr\omega^2$$

$$m_c = 3.5\ m = 3.5 \times 3 = \underline{10.5\ \text{kg}}$$

14.14 BALANCING MACHINES

Though care is taken in the design of rotating parts of a machine to eliminate any out-of-balance force or couple, still some residual unbalance will always be left in the finished part. This may happen due to slight variation in the density of the material or inaccuracies in the casting or machining. Since the centrifugal force and couple vary as the square of the speed, even the small errors may lead to serious troubles at high speeds of rotation. Thus, effort is made to measure these out-of balance forces and couples so that suitable corrections can be made to the part to reduce the final errors. The machines used may be to measure the static unbalance or dynamic unbalance or both.

A *balancing machine* is able to indicate whether a part is in balance or not and if it is not, then it measures the unbalance by indicating its magnitude and location.

1. Static Balancing Machines

Static balancing machines are helpful for parts of small axial dimensions such as fans, gears and impellers, etc., in which the mass lies practically in a single plane.

(i) Figure 14.45 shows a simple kind of static balancing machine. The machine is of the form of a weighing machine. One arm of the machine has a mandrel to support the part to be balanced and the other arm supports a suspended deadweight to make the beam approximately horizontal. The mandrel is then rotated slowly either by hand or by a motor. As the mandrel is rotated, the beam will oscillate depending upon the unbalance of the part. If the unbalance is represented by a mass m at radius r, the apparent weight is greatest when m is at the position

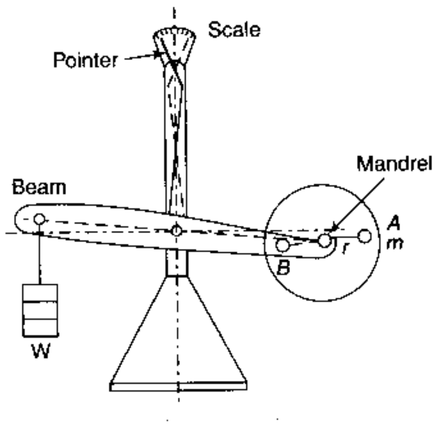


Fig. 14.45

A and least when it is at *B* as the lengths of the arms in the two cases will be maximum and minimum. A calibrated scale along with the pointer can also be used to measure the amount of unbalance. Obviously, the pointer remains stationary in case the body is statically balanced.

(ii) A more sensitive machine than the previous one is shown in Fig. 14.46. It consists of a cradle supported on two pivots *P-P* parallel to the axis of rotation of the part and held in position by two springs *S-S*. The part to be tested is mounted on the cradle and is flexibly coupled to an electric motor. The motor is started and the speed of rotation is adjusted so that it coincides with the natural frequency of the system. Thus, the condition of resonance is obtained under which even a small amount of unbalance generates large amplitude of the cradle.

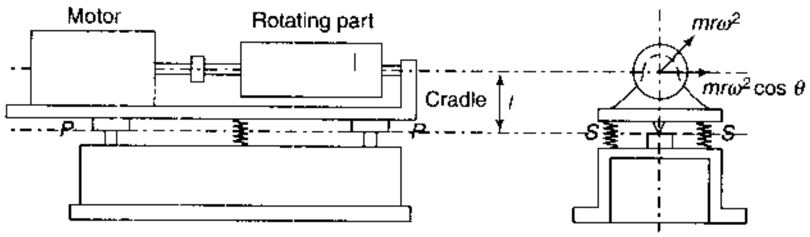


Fig. 14.46

The moment due to unbalance = $(mr\,\omega^2.\cos\theta).l$ where ω is the angular velocity of rotation. Its maximum value is $mr\,\omega^2.l$. If the part is in static balance but dynamic unbalance, no oscillation of the cradle will be there as the pivots are parallel to the axis of rotation.

2. Dynamic Balancing Machines

For dynamic balancing of a rotor, two balancing or countermasses are required to be used in any two convenient planes. This implies that the complete unbalance of any rotor system can be represented by two unbalances in those two planes. Balancing is achieved by addition or removal of masses in these two planes, whichever is convenient. The following is a common type of dynamic balancing machine.

Pivoted-cradle Balancing Machine In this type of machine, the rotor to be balanced is mounted on half-bearings in a rigid carriage and is rotated by a drive motor through a universal joint (Fig.14.47). Two balancing planes *A* and *B* are chosen on the rotor. The cradle is provided with pivots on left and right sides of the rotor which are purposely adjusted to coincide with the two correction planes. Also the pivots can be put in the locked or unlocked position. Thus, if the left pivot is released, the cradle and the specimen are free to oscillate about the locked (right) pivot. At each end of the cradle, adjustable springs and dashpots are provided to have a single degree of freedom system. Usually, their natural frequency is tuned to the motor speed.

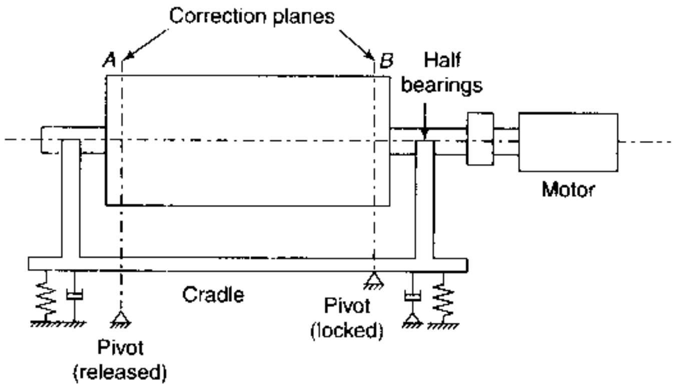


Fig. 14.47

The following procedure is adopted for testing:

1. First, either of the two pivots say left is locked so that the readings of the amount and the angle of location of the correction in the right hand plane can be taken. These readings will be independent of any unbalance in the locked plane as it will have no moment about the fixed pivot.

2. A trial mass at a known radius is then attached to the right-hand plane and the amplitude of oscillation of the cradle is noted.
3. The procedure is repeated at various angular positions with the same trial mass.
4. A graph is then plotted of amplitude vs. angular positions of the trial mass to know the optimum angular position for which amplitude is minimum. Then at this position, the magnitude of the trial mass is varied and the exact amount is found by trial and error which reduces the unbalance to almost zero.
5. After obtaining the unbalance in one plane, the cradle is locked in the right-hand pivot and released in the left-hand pivot. The above procedure is repeated to obtain the exact balancing mass required in that plane.
6. Usually, a large number of test runs are required to determine the exact balance masses in this type of machine. However, by adopting the following procedure, the balance masses can be obtained by making only four test runs.

First, make a test run without attaching any trial mass and note down the amplitude of the cradle vibrations. Then attach a trial mass m at some angular position and note down the amplitude of the cradle vibrations by moving the rotor at the same speed. Next detach the trial mass and again attach it at a 90° angular position relative to the first position at the same radial distance. Note down the amplitude by rotating the rotor at the same speed. Take the last reading in the same manner by fixing the trial mass at 180°. Let the four readings be

Trial mass	*Amplitude*
0	X_1
m at 0°	X_2
m at 90°	X_3
m at 180°	X_4

Make the following construction (Fig. 14.48):

Draw a triangle OBE by taking $OE = 2X_1$, $OB = X_2$ and $BE = X_4$. Mark the mid-point A on OE. Join AB.

Now,

$$\mathbf{OB} = \mathbf{OA} + \mathbf{AB}$$

where

OB = Effect of unbalance mass + Effect of the trial mass at 0°.

OA = Effect of unbalanced mass

Thus, **AB** represents the effect of the attached mass at 0°. The proof is as follows:

Extend BA to D such that $AD = AB$. Join OD and DE.

Now when the mass m is attached at 180° at the same radial distance and speed, the effect must be equal and opposite to the effect at 0°, i.e., if **AB** represents the effect of the attached mass at 0°, **AD** represents the effect of the attached mass at 180°.

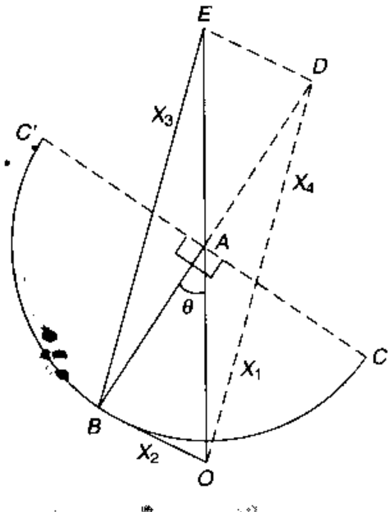


Fig. 14.48

Since

$$\mathbf{OD} = \mathbf{OA} + \mathbf{AD}$$

OD must represent the combined effect of unbalance mass and the effect of the trial mass at 180° ($X4$)

Now, as the diagonals of the quadrilateral $OBED$ bisect each other at A, it is a parallelogram which means

BE is parallel and equal to OD. Thus, BE also represents the combined effect of unbalance mass and the effect of the trial mass at 180° or $X4$ which is true as it is made in the construction.

Now as OA represents the unbalance, the correction has to be equal and opposite of it or AO. Thus, the correction mass is given by

$$\frac{m_c}{m} = \frac{OA}{AB}$$

or $$m_c = m.\frac{OA}{AB} \text{ at an angle } \theta \text{ from the second reading at } 0^\circ.$$

For the correction of the unbalance, the mass mc has to be put in the proper direction relative to AB which may be found by considering the reading X_3.

Draw a circle with A as centre and AB as the radius. As the trial mass as well as the speed of the test run at 90° is the same, the magnitude must be equal to AB or AD, and AC or AC' must represent the effect of the trial mass. If OC represents X_3 then angle is opposite to the direction of angle measurement. If OC' represents X_3 then angle measurement is taken in the same direction.

Example 14.25 *During the balancing of a rotor using a trial mass of 600 g, the four readings of the amplitude of the cradle taken are as follows:*

Trial mass	*Amplitude*
0	6.2 mm (X_1)
at 0°	9.8 mm (X_2)
at 90°	15.0 mm (X_3)
at 180°	12.4 mm (X_4)

Find the magnitude and location of the correction mass to balance the rotor.

Solution Draw a triangle OBE by taking $OE = 2X_1$, $OB = X_2$ and $BE = X_4$. Mark the mid-point A on OE. Join AB (Fig. 14.49). On measurement, AB = 9.3 mm and $\theta = 75°$.

Then

$$m_c = m.\frac{OA}{AB} = 600 \times \frac{6.2}{9.3} = 400 \text{ g}$$

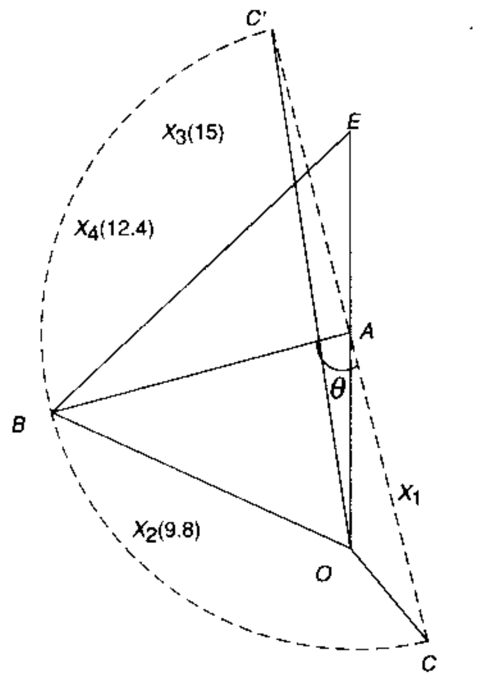


Fig. 14.49

As X_3 is found to be equal to OC' which means the readings are taken clockwise and since for complete balancing AB should merge with AO, the mass is attached at 75° counter-clockwise from the direction of the second reading.

14.15 FIELD BALANCING

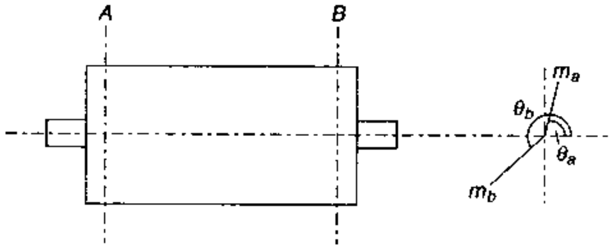


Fig. 14.50

In heavy machinery like turbines and generators, it is not possible to balance the rotors by mounting them in the balancing machines. In such cases, the balancing has to be done under normal conditions on its own bearings. Assume the two balancing planes of a rotor to be A and B (Fig. 14.50).

1. First, the rotor is rotated at a speed which provides measurable amplitudes at planes A and B. Let the vectors **A** and **B** represent the amplitudes due to the unbalance of the rotor in planes A and B respectively.
2. Attach a trial mass m_a in the plane A at a known radius and known angular position and run the rotor at the same speed as in the first case. Measure the amplitudes in the two planes A and B. Let $\mathbf{A_1}$ and $\mathbf{B_1}$ represent the amplitudes of the rotor in planes A and B respectively. Thus

   Effect at A of the unbalance + Effect at A of trial mass in plane A = $\mathbf{A_1}$

   ∴ Effect at A of trial mass in plane $A = \mathbf{A_1} - \mathbf{A}$

   Effect at B of the unbalance + Effect at B of trial mass in plane $A = \mathbf{B_1}$

   ∴ Effect at B of trial mass in plane $A = \mathbf{B_1} - \mathbf{B}$
3. Make a third run of the rotor by attaching a trial mass m_b in plane B at a known radius and known angular position and run the rotor at the same speed as in the first two cases. Measure the amplitudes in the two planes A and B. Let $\mathbf{A_2}$ and $\mathbf{B_2}$ represent the amplitudes of the rotor in planes A and B respectively. Thus

   Effect at A of the unbalance + Effect at A of trial mass in plane $B = \mathbf{A_2}$

   ∴ Effect at A of trial mass in plane $B = \mathbf{A_2} - \mathbf{A}$

   Effect at B of the unbalance + Effect at B of trial mass in plane $B = \mathbf{B_2}$

   ∴ Effect at B of trial mass in plane $B = \mathbf{B_2} - \mathbf{B}$

Let m_{ca} and m_{cb} be the counter or balancing masses in planes A and B respectively placed at the same radii as the trial masses.

Let $m_{ca} = \alpha \times m_a$ and $m_{cb} = \beta \times m_b$

where $\alpha = a.e^{i\theta_a}$, i.e., the countermass in the plane A is a times the trial mass located at an angle θ_a with its direction.

and $\beta = b.e^{i\theta_b}$, i.e., the countermass in plane B is b times the trial mass located at an angle θ_b with its direction.

For complete balancing of the rotor, the effect of the balancing masses must be to nullify the unbalance in the two planes, i.e., in the plane A it must be equal to −**A** and in plane B equal to −**B**.

Thus

$$\alpha(\boldsymbol{A_1} - \boldsymbol{A}) + \beta(\boldsymbol{A_2} - \boldsymbol{A}) = -\boldsymbol{A} \quad \text{(i)}$$

and

$$\alpha(\boldsymbol{B_1} - \boldsymbol{B}) + \beta(\boldsymbol{B_2} - \boldsymbol{B}) = -\boldsymbol{B} \quad \text{(ii)}$$

These equations can be solved for α and β. Multiplying (i) with $(\boldsymbol{B_2} - \boldsymbol{B})$ and (ii) with $(\boldsymbol{A_2} - \boldsymbol{A})$,

$$\alpha(\boldsymbol{A_1} - \boldsymbol{A})(\boldsymbol{B_2} - \boldsymbol{B}) + \beta(\boldsymbol{A_2} - \boldsymbol{A})(\boldsymbol{B_2} - \boldsymbol{B}) = -\boldsymbol{A}(\boldsymbol{B_2} - \boldsymbol{B}) \quad \text{(iii)}$$

$$\alpha(\boldsymbol{A_2} - \boldsymbol{A})(\boldsymbol{B_1} - \boldsymbol{B}) + \beta(\boldsymbol{A_2} - \boldsymbol{A})(\boldsymbol{B_2} - \boldsymbol{B}) = -\boldsymbol{B}(\boldsymbol{A_2} - \boldsymbol{A}) \quad \text{(iv)}$$

Subtracting (iv) from (iii),

$$\alpha\,[(A_1 - A)(B_2 - B) - (A_2 - A)(B_1 - B)] = B(A_2 - A) - A(B_2 - B)$$

or
$$\alpha = \frac{B(A_2 - A) - A(B_2 - B)}{(A_1 - A)(B_2 - B) - (A_2 - A)(B_1 - B)} \tag{14.42}$$

Multiplying (i) with $(B_1 - B)$ and (ii) with $(A_1 - A)$,

$$\alpha\,(A_1 - A)(B_1 - B) + \beta\,(A_2 - A)(B_1 - B) = -A(B_1 - B) \tag{v}$$

$$\alpha\,(A_1 - A)(B_1 - B) + \beta\,(A_1 - A)(B_2 - B) = -B(A_1 - A) \tag{vi}$$

Subtracting (v) from (vi),

$$\beta\,[(A_1 - A)(B_2 - B) - (A_2 - A)(B_1 - B)] = -B(A_1 - A) + A(B_1 - B)$$

or
$$\beta = \frac{A(B_1 - B) - B(A_1 - A)}{(A_1 - A)(B_2 - B) - (A_2 - A)(B_1 - B)} \tag{14.43}$$

Example 14.26 *While balancing a turbine rotor by the field balancing technique, the results are obtained as shown in Table 14.7. Find the correct balance masses to be placed in planes A and B at the same radii as for the trial masses. Also, find the angular positions of the balance masses with respect to trial masses to have the complete dynamic balance of the rotor.*

Solution For the sake of simplicity, the multiplier 10^{-3} in the vectors A, A_1, A_2 and B, B_1, B_2 have been omitted which does not affect the end result.

As $e^{j\theta} = \cos\theta + j\sin\theta$

$\mathbf{A} = 2.5\angle 20° = 2.5(\cos 20° + j\sin 20°) = 2.349 + 0.855j$

$\mathbf{A_1} = 4.2(\cos 100° + j\sin 100°) = -0.729 + 4.136j$

$\mathbf{A_2} = 3.6(\cos 55° + j\sin 55°) = -2.065 - 2.949j$

$\mathbf{B} = 4.5\angle 60° = 4.5(\cos 60° + j\sin 60°) = 2.25 + 3.897j$

$\mathbf{B_1} = 3.4(\cos 125° + j\sin 125°) = -1.95 + 2.785j$

$\mathbf{B_2} = 2.6(\cos 210° + j\sin 210°) = -2.25 - 1.3j$

$\mathbf{A_1} - \mathbf{A} = -0.729 + 4.136j - (2.349 + 0.855j)$
$= -3.078 + 3.281j = 4.5\,e^{j(133.2°)}$

Similarly, $\mathbf{A_2} - \mathbf{A} = -0.284 + 2.094j = 2.113\,e^{j(97.7°)}$

$\mathbf{B_1} - \mathbf{B} = -4.2 - 1.112j = 4.345\,e^{j(194.8°)}$

$\mathbf{B_2} - \mathbf{B} = -4.5 - j5.197 = 6.875\,e^{j(229.1°)}$

or writing the vectors in the polar mode and using the complex mode of the calculator.

$\mathbf{A} = 2.5\angle 20°$; $\mathbf{A_1} = 4.2\angle 100°$; $\mathbf{A_2} = 3.6\angle 55°$

$\mathbf{B} = 4.5\angle 60°$; $\mathbf{B_1} = 3.4\angle 125°$; $\mathbf{B_2} = 2.6\angle 210°$

$\mathbf{A_1} - \mathbf{A} = 4.5\angle 133.2°$; $\mathbf{A_2} - \mathbf{A} = 2.113\angle 97.7°$;

$\mathbf{B_1} - \mathbf{B} = 4.345\angle 194.8°$; $\mathbf{B_2} - \mathbf{B} = 6.875\angle 229.1°$

These values of vector differences can also be obtained graphically as shown in Fig. 14.51 (a) and (b).

Table 14.7

		Plane A		Plane B	
No.	*Trial mass (kg)*	*Amplitude (mm)*	*Phase angle (degrees)*	*Amplitude (mm)*	*Phase angle (degrees)*
1.	0	2.5×10^{-3}	20	4.5×10^{-3}	60
2.	3 (in plane *A*)	4.2×10^{-3}	100	3.4×10^{-3}	125
3.	3(in plane *B*)	3.6×10^{-3}	55	2.6×10^{-3}	210

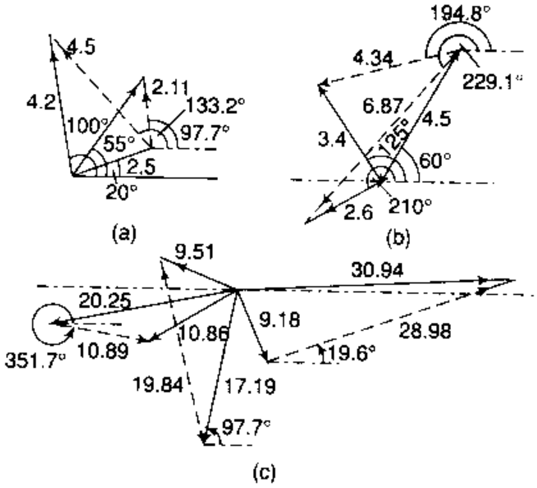


Fig. 14.51

Now, $\alpha = \dfrac{B(A_2 - A) - A(B_2 - B)}{(A_1 - A)(B_2 - B) - (A_2 - A)(B_1 - B)}$ [Eq. 14.42]

or $\alpha = \dfrac{\begin{bmatrix} 4.5e^{i(60^\circ)} \times 2.113e^{i(97.7^\circ)} \\ -2.5e^{i(20^\circ)} \times 6.875e^{i(229.1^\circ)} \end{bmatrix}}{\begin{bmatrix} 4.5e^{i(133.2^\circ)} \times 6.875e^{i(229.1^\circ)} \\ -2.113e^{i(97.7^\circ)} \times 4.345e^{i(194.8^\circ)} \end{bmatrix}}$

$= \dfrac{9.51e^{i(157.7^\circ)} - 17.188e^{i(249.1^\circ)}}{30.94e^{i(2.3^\circ)} - 9.18e^{i(292.5^\circ)}}$

The numerator and the denominator can be solved analytically or graphically [Fig. 14.51(c)].

i.e.,

$$\alpha = \frac{19.84e^{i(97.7^\circ)}}{28.98e^{i(19.6^\circ)}} = 0.685\, e^{i(78.1^\circ)}$$

Similarly,

$$\beta = \frac{A(B_1 - B) - B(A_1 - A)}{(A_1 - A)(B_2 - B) - (A_2 - A)(B_1 - B)}$$ [Eq. 14.43)]

or $\beta = \dfrac{\begin{bmatrix} 2.5e^{i(20^\circ)} \times 4.345e^{i(194.8^\circ)} \\ -4.5e^{i(60^\circ)} \times 4.5e^{i(133.2^\circ)} \end{bmatrix}}{\begin{bmatrix} 4.5e^{i(133.2^\circ)} \times 6.875e^{i(229.1^\circ)} \\ -2.113e^{i(97.7^\circ)} \times 4.345e^{i(194.8^\circ)} \end{bmatrix}}$

$= \dfrac{10.86e^{i(214.8^\circ)} - 20.25e^{i(193.2^\circ)}}{30.94e^{i(2.3^\circ)} - 9.18e^{i(292.5^\circ)}}$

$= \dfrac{10.895e^{i(351.7^\circ)}}{28.98e^{i(19.6^\circ)}} = 0.376\, e^{i(332.1^\circ)}$

Thus, the balance mass in the plane A
$= 0.685 \times 3 = 2.055$ kg

Angular position = 78.1° counter-clockwise with the direction of trial mass in the plane A.

Similarly, the balance mass in plane B
$= 0.376 \times 3 = 1.128$ kg

Angular position = 332.1° counter-clockwise with the direction of trial mass in the plane B.

Summary

1. A system of rotating masses is said to be in *static balance* if the combined mass centre of the system lies on the axis of rotation.
2. Several masses rotating in different planes are said to be in *dynamic balance* when there does not exist any resultant centrifugal force as well as the resultant couple.
3. Balancing of a *linkage* implies that the total centre of its mass remains stationary so that the vector sum of all the frame forces always remains zero.
4. *Primary accelerating force* in a reciprocating engine is $mr\omega^2 \cos\theta$ along the line of stroke.
5. *Secondary accelerating force* in a reciprocating engine is $mr\omega^2 \cos(2\theta)/n$ along the line of stroke.
6. In reciprocating engines, unbalanced forces along the line of stroke are more harmful than the forces perpendicular to the line of stroke.
7. In locomotives, *hammer-blow* is the maximum vertical unbalanced force caused by the mass to balance the reciprocating masses and *swaying couple* tends to make the leading wheels sway from side to side due to unbalanced primary forces along the lines of stroke.
8. The effect of the secondary force is equivalent

to an imaginary crank of length $r/4n$ rotating at double the angular velocity, i.e., twice of the engine speed.

9. For complete balancing of the reciprocating parts, the primary forces and primary couples as well as the secondary forces and secondary couples must balance.
10. If a reciprocating mass is transferred to the crank pin, the axial component of the resulting centrifugal force along the cylinder axis is the primary unbalanced force.
11. A six-cylinder four-stroke engine is a completely balanced engine.
12. In V-engines, a common crank is operated by two connecting rods at some angle.
13. In radial engines with a number of connecting rods and a common crank, the analysis is much simplified by using the method of *direct and reverse cranks*.
14. A *balancing machine* is able to indicate whether a part is in balance or not and if it is not, then it measures the unbalance by indicating its magnitude and location.
15. *Field balancing* is adopted in heavy machinery like turbines and generators where it is not possible to balance the rotors by mounting them on the balancing machines.

Exercises

1. Why is balancing necessary for rotors of high-speed engines?
2. What is meant by static and dynamic unbalance in machinery? How can the balancing be done?
3. Two masses in different planes are necessary to rectify the dynamic unbalance. Comment.
4. Explain the method of finding the countermasses in two planes to balance the dynamic unbalance of rotating masses.
5. What do you mean by force balancing of linkages? How is it achieved? Explain.
6. What do you mean by primary and secondary unbalance in reciprocating engines?
7. Deduce expressions for variation in tractive force, swaying couple and hammer blow for an uncoupled two cylinder locomotive engine.
8. Determine the unbalanced forces and couples in case of following in-line engines:
   (i) two-cylinder engine
   (ii) four-cylinder four-stroke engine
   (iii) six-clinder four-stroke engine.
9. Find the magnitudes of the unbalanced primary and secondary forces in V-engines. Deduce the expressions when the lines of stroke of the two cylinders are at 60° and 90° to each other.
10. Explain the method of direct and reverse cranks to determine the unbalance forces in radial engines.
11. What do you mean by balancing machines? Describe any one type of a static balancing machine.
12. Describe the function of a pivoted-cradle balancing machine with the help of a neat sketch. Show that it is possible to make only four test runs to obtain the balance masses in such a machine.
13. What is field balancing of rotors? Explain the procedure.
14. The rotor shown in Fig. 14.2(a) has the following properties:

    $m_1 = 3$ kg $\quad r_1 = 30$ mm $\quad \theta_1 = 30°$
    $m_2 = 4$ kg $\quad r_2 = 20$ mm $\quad \theta_2 = 120°$
    $m_3 = 2$ kg $\quad r_3 = 25$ mm $\quad \theta_3 = 270°$

    Find the amount of the countermass of a radial distance of 35 mm for the static balance.

    (2.13 kg; 239.4°)
15. The rotor shown in Fig. 14.6(a) has the following properties:

    $m_1 = 3$ kg $\quad r_1 = 30$ mm $\quad \theta_1 = 30° \quad l_1 = 100$ mm
    $m_2 = 4$ kg $\quad r_2 = 20$ mm $\quad \theta_2 = 120° \quad l_2 = 300$ mm
    $m_3 = 2$ kg $\quad r_3 = 25$ mm $\quad \theta_3 = 270° \quad l_3 = 600$ mm
    $r_{c1} = 35$ mm and $r_{c2} = 20$ mm

    l_1, l_2 and l_3 are the distances from the bearing 1. The axial distance between the bearings is 500 mm. Determine the countermass to be placed in the places of m_1 and a mid-plane of m_2 and m_3 for the complete balance.

    ($m_{c1} = 1.96$ kg, 54.3°; $m_{c2} = 3.25$ kg, 238.2°)
16. A rotor has the following properties:

Mass	Magnitude	Radius	Angle	Axial distance from 1st mass
1	9 kg	100 mm	0°	
2	7 kg	120 mm	60°	160 mm
3	8 kg	140 mm	135°	320 mm
4	6 kg	120 mm	270°	560 mm

If the shaft is balanced by two countermasses located at 100 mm radii and revolving in planes midway of planes 1 and 2, and midway of 3 and 4, determine the magnitude of the masses and their respective angular positions.

(6.9 kg, 23°; 15.8 kg, 222.6°)

17. Four masses *A*, *B*, *C* and *D* are completely balanced. Masses *C* and *D* make angles of 90° and 210° respectively with *B* in the same sense. The planes containing *B* and *C* are 300 mm apart. Masses *A*, *B*, *C* and *D* can be assumed to be concentrated at radii of 360, 480, 240 and 300 mm respectively. The masses *B*, *C* and *D* are 15 kg, 25 kg and 20 kg respectively. Determine the
    (i) mass *A* and its angular position
    (ii) positions of planes *A* and *D*

(10 kg, 236°; A is 985 mm towards right and *D* is 378 mm towards left of plane B)

18. A single-cylinder reciprocating engine has a reciprocating mass of 60 kg. The crank rotates at 60 rpm and the stroke is 320 mm. The mass of the revolving parts at 160 mm radius is 40 kg. If two-thirds of the reciprocating parts and the whole of the reciprocating parts and the whole of the revolving parts are to be balanced, determine the
    (i) balance mass required at a radius of 350 mm
    (ii) unbalanced force when the crank has turned 50° from the top-dead centre

(36.57 kg; 209.9 N)

19. The cranks of a three-cylinder locomotive are set at 120°. The reciprocating masses are 450 kg for the cylinder and 390 kg for each outside cylinder. The pitch of the cylinders is 1.2 m and the stroke of each piston is 500 mm. The planes of rotation of the balance masses are 960 mm from the inside cylinder. If 40% of the reciprocating masses are to be balanced, determine the magnitude and the position of the balancing masses required at a radial distance of 500 mm, and the hammer-blow per wheel when the axle rotates at 350 rpm.

(86.25 kg each at 24° and 216°; 57.9 kN)

20. The firing order of a six-cylinder vertical four-stroke in-line engine is 142635. The piston stroke is 80 mm and the length of each connecting rod is 180 mm. The pitch distances between the cylinder centre lines are 80 mm, 80 mm, 120 mm, 80 mm and 80 mm respectively. The reciprocating mass per cylinder is 1.2 kg and the engine speed is 2400 rpm. Determine the out of balance primary and secondary forces and couples on the engine taking a plane midway between the cylinders 3 and 4 as the reference plane.

(Completely balanced engine; no out of balance primary and secondary forces and couples)

21. A four-cylinder engine is arranged as shown in Fig. 14.22. The reciprocating masses in planes 1 and 4 are each 142 kg and in planes 2 and 3 are each 200 kg. If the crank radii are 400 mm each, the speed 200 rpm and the length of the connecting rod is 1.6 m, determine the magnitude of primary and secondary forces and couples. Given that $\alpha = 25°$, $\beta = 50°$, $l_1 = 1.28$ m and $l_2 = 0.5$ m.

(Primary forces and couples are zero; Secondary force = 4959 N; Secondary couple = 20.85 kN.m)

22. The cylinders of a V-engine are set at an angle of 40° with both cylinders connected to a common crank. The connecting rod is 300 mm long and the crank radius is 60 mm. The reciprocating mass is 1 kg per cylinder whereas the rotating mass at the crank pin is 1.5 kg. A balance mass equivalent to 1.8 kg is also fitted opposite to the crank at a radius of 80 mm. Determine the maximum and the minimum values of the primary and secondary forces due to inertia of the reciprocating and rotating masses if the engine rotates at 900 rpm.

(461.4 N, 354.9 N; 153.4 N, 46.9 N)

23. Two outer cranks of a four-crank engine are set at 120° to each other with each reciprocating mass as 400 kg. The spacing between the planes of rotation of adjacent cranks are 450 mm, 750 mm and 600 mm. Determine the reciprocating mass and the relative angular position of each of the inner cranks if the engine is to be in complete primary balance. Also, determine the maximum secondary unbalanced force if the length of the crank and the connecting rod are 300 mm and 1200 mm respectively and the speed is 240 rpm.

(878 kg, 314°; 853 kg, 162°, 90 kN)

24. Each crank of a four-cylinder vertical engine is 225 mm. The reciprocating masses of the first, second and the fourth cranks are 100 kg, 120 kg and 100 kg and the planes of rotation are 600 mm, 300 mm and 300 mm from the plane of rotation of the third crank. Determine the mass of the reciprocating parts of the third cylinder and the relative angular positions of the cranks if the engine is in complete primary balance.

(120 kg; $\theta_1 = 0°$, $\theta_2 = 157.7°$, $\theta_3 = 229.5°$, $\theta_4 = 27.2°$)

25. The connecting rods of a three-cylinder air compressor are coupled to a single crank and the axes are at 120° to one another. Each connecting rod is 180 mm long and the stroke is 120 mm. The reciprocating parts have a mass of 1.8 kg per

cylinder. Find the magnitude of the primary and secondary forces when the engine runs at 1200 rpm.

(2.558 kN, 852.7 N)

26. A radial aero-engine has nine cylinders equally spaced with all the connecting rods coupled to a common crank. The crank and each of the connecting rods are 140 mm and 540 mm respectively. The reciprocating mass per cylinder is 2.4 kg. Determine the magnitude and the angular position of the balance masses required at the crank radius for complete primary and secondary balancing of the engine. (10.8 kg)

27. While balancing a turbine rotor by the field balancing technique, the results obtained are shown in Table 14.8.

Find the correct balance masses to be placed in planes *A* and *B* at the same radii as for the trial masses. Also, find the angular positions of the balance masses with respect to trial masses to have the complete dynamic balance of the rotor.

(2.62 kg, 71.3°; 1.304 kg, 340.8°)

Table 14.8

		Plane A		*Plane B*	
No.	***Trial mass (Kg)***	*Amplitude (mm)*	***Phase angle (degrees)***	*Amplitude (mm)*	*Phase angle (degrees)*
1	**0**	3×10^{-3}	**25**	5×10^{-3}	70
2	**3 (in plane *A*)**	4.5×10^{-3}	110	3.8×10^{-3}	135
3	**3 (in plane *B*)**	4×10^{-3}	**60**	3.2×10^{-3}	**215**

15 BRAKES AND DYNAMOMETERS

Introduction

A *brake* is an appliance used to apply frictional resistance to a moving body to stop or retard it by absorbing its kinetic energy. In general, in all types of motion, there is always some amount of resistance which retards the motion and is sufficient to bring the body to rest. However, the time taken and the distance covered in this process is usually too large. By providing brakes, the external resistance is considerably increased and the period of retardation shortened.

A *dynamometer* is a brake incorporating a device to measure the frictional resistance applied. This is used to determine the power developed by the machine, while maintaining its speed at the rated value.

The functional difference between a clutch and a brake is that a clutch connects two moving members of a machine whereas a brake connects a moving member to a stationary member.

15.1 TYPES OF BRAKES

The following are the main types of mechanical brakes:

(i) Block or shoe brake
(ii) Band brake
(iii) Band and block brake
(iv) Internal expanding shoe brake

15.2 BLOCK OR SHOE BRAKE

A block or shoe brake consists of a block or shoe which is pressed against a rotating drum. The force on the drum is increased by using a lever [Fig. 15.1(a)]. If only one block is used for the purpose, a side thrust on the bearing of the shaft supporting the drum will act. This can be prevented by using two blocks on the two sides of the drum [Fig.15.1(b)]. This also doubles the braking torque.

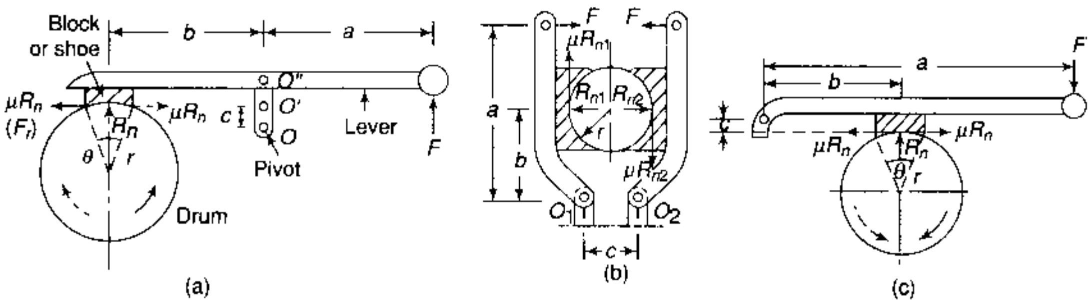


Fig. 15.1

A material softer than that of the drum or the rim of the wheel is used to make the blocks so that these can be replaced easily on wearing. Wood and rubber are used for light and slow vehicles and cast steel for heavy and fast ones.

Let r = radius of the drum

μ = coefficient of friction

F_r = radial force applied on the drum (not shown in the figure)

R_n = normal reaction on the block ($= F_r$)

F = force applied at the lever end

F_f = frictional force = μR_n

Assuming that the normal reaction R_n and frictional force F_f act at the mid-point of the block,

Braking torque on the drum = frictional force × radius

or

$$T_B = \mu R_n \times r \tag{15.1}$$

To obtain R_n, consider the equilibrium of the block as follows.

The direction of the frictional force on the drum is to be opposite to that of its rotation while on the block it is in the same direction. Taking moments about the pivot O [Fig. 15.1(a)],

$$F \times a - R_n \times b + \mu R_n \times c = 0$$

$$R_n = \frac{Fa}{b - \mu c} \tag{15.2}$$

Also

$$F = R_n \frac{b - \mu c}{a} \tag{15.3}$$

- When $b = \mu c$, $F = 0$, which implies that the force needed to apply the brake is virtually zero, or that once contact is made between the block and the drum, the brake is applied itself. Such a brake is known as a *self-locking brake*.
- As the moment of the force F_f about O is in the same direction as that of the applied force F, F_f aids in applying the brake. Such a brake is known as a *self-energised brake*.
- If the rotation of the drum is reversed, i.e., it is made clockwise,

  $$F = R_n [(b + \mu c)/a]$$

  which shows that the required force F will be far greater than what it would be when the drum rotates counter-clockwise.
- If the pivot lies on the line of action of F_f, i.e., at O', $c = 0$ and $F = R_n \frac{a}{b}$, which is valid for clockwise as well as for counter-clockwise rotation.
- If c is made negative, i.e., if the pivot is at O'',

  $$F = R_n \left(\frac{b + \mu c}{a} \right) \text{ for counter-clockwise rotation}$$

  and

  $$F = R_n \left(\frac{b - \mu c}{a} \right) \text{ for clockwise rotation}$$

- In case the pivot is provided on the same side of the applied force and the block as shown in Fig. 15.1(c), the equilibrium condition can be considered accordingly.

In the above treatment, it is assumed that the normal reaction and the frictional force act at the mid-point of the block. However, this is true only for small angles of contact. When the angle of contact is more than 40Y, the normal pressure is less at the ends than at the centre. In that case, μ has to be replaced by an equivalent coefficient of friction μ' given by

$$\mu' = \mu\left(\frac{4\sin\left(\frac{\theta}{2}\right)}{\theta + \sin\theta}\right)$$

Example 15.1 *Two block brakes are shown in Figs. 15.2(a) and (b). The diameter of the brake drum in each case is 1 m. Each brake sustains 240 N.m of torque at 400 rpm. The coefficient of friction is 0.32. Determine the required force to be applied when the angle of contact in the two cases are 35° and 100°. Also, find the new values of c for self-locking of the brake.*

Assume the rotation of the drum to be both clockwise and counter-clockwise.

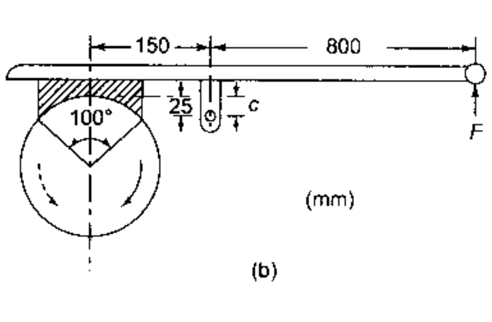


Fig. 15.2

Solution

$T_B = 240$ N.m, $\quad r = 0.5$ m

(a) *Angle of contact = 35°.*

$\mu = 0.32$

$T_B = \mu R_n r$

$240 = 0.32 \times R_n \times 0.5$

$R_n = 1500$ N

Rotation clockwise

$F.\,a - R_n b - \mu R_n c = 0$

$F \times 0.8 - 1500 \times 0.15 - 0.32 \times 1500 \times 0.025 = 0$

$F \times 0.8 - 225 - 12 = 0$

$F = \underline{296.25 \text{ N}}$

Rotation counter-clockwise

$F \times 0.8 - 1500 \times 0.15 + 32 \times 1500 \times 0.025 = 0$

$F = \underline{266.25 \text{ N}}$

For self-locking, F is to be zero. For a positive value of c this is possible for counter-clockwise rotation of the drum, i.e., when

$$0 - 1500 \times 0.15 + 0.32 \times 1500 \times c = 0$$

or $\quad c = \dfrac{0.15}{0.32} = 0.469$ m or $\underline{469 \text{ mm}}$

(b) Angle of contact = 100°

$$\mu' = \mu\left(\frac{4\sin(\theta/2)}{\theta + \sin\theta}\right)$$

$$= 0.32\left(\frac{4\sin 50°}{100 \times \dfrac{\pi}{180} + \sin 100°}\right) = 0.36$$

$T_B = \mu' R_n r$

$240 = 0.36 \times R_n \times 0.5$

$R_n = 1333$ N

Rotation clockwise

$F \times 0.8 \quad 1333 \times 0.15 - 0.36 \times 1333 \times 0.025 = 0$

$0.8\,F - 200 \quad 12 = 0$

$F = \underline{265\text{ N}}$

Rotation counter-clockwise

$F \times 0.8 - 1333 \times 0.15 + 0.36 \times 1333 \times 0.025 = 0$

$0.8\,F - 200 + 12 = 0$

$F = \underline{235\text{ N}}$

For self-locking

$0 - 1333 \times 0.15 + 0.36 \times 1333 \times c = 0$

or $c = \dfrac{0.15}{0.36} = 0.417$ m or $\underline{417\text{ mm}}$

(*Note*: 400 rpm is the superfluous data in the problem)

Example 15.2 *A bicycle and rider, travelling at 12 km/h on a level road, have a mass of 105 kg. A brake is applied to the rear wheel which is 800 mm in diameter. The pressure on the brake is 80 N and the coefficient of friction is 0.06. Find the distance covered by the bicycle and number of turns of its wheel before coming to rest.*

Solution

$m = 105$ kg $\qquad d = 0.8$ m

$v = \dfrac{12000}{3600} = 3.333$ m/s $\qquad F_r = 80\text{ N} = R_n$

$\mu = 0.06$

Let s = distance covered by the bicycle before it comes to rest.

Work done against friction = KE of the bicycle and the rider

$$\mu R_n s = \frac{1}{2} m v^2$$

$$0.06 \times 80 \times s = \frac{1}{2} \times 105 \times (3.333)^2$$

$s = \underline{121.5\text{ m}}$

$\pi\,d\,n = s$

or $\pi \times 0.8 \times n = 121.5$

$n = 48.3$ revolutions

Example 15.3 *A brake drum of 440 mm in diameter is used in a braking system as shown in Fig. 15.3(a). The brake lever is inclined at an angle of 20° with the horizontal. A vertical force of 400-N magnitude is applied at the lever end. The coefficient of friction is 0.35. The brake drum has a mass of 160 kg and it rotates at 1500 rpm. Determine the*

(i) braking torque

(ii) number of revolutions made by the drum and the time taken before coming to rest from the instant the brake is applied

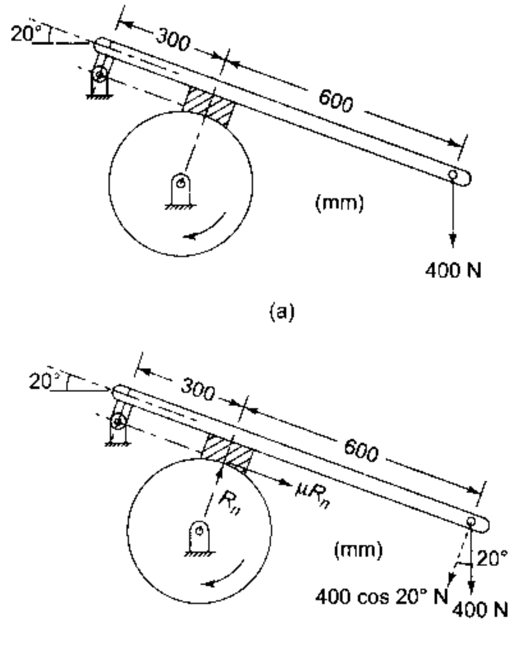


Fig. 15.3

Solution

$d = 440$ mm, $r = 220$ mm, $\mu = 0.35$, $m = 160$ kg, $N = 1500$ rpm, $F = 400 \cos 20°$N

Angle of contact is not given. It may be assumed small so that $\mu = 0.35$

The line of frictional force passes through the fulcrum.

(i) Taking moments about the fulcrum [Fig. 15.3(b)].

$$400 \cos 20° \times 900 + \mu R_n \times 0 - R_n \times 300 = 0$$

or $R_n = 1127.6$ N

$$T_B = \mu R_n r = 0.35 \times 1127.6 \times 0.22 = 86.8\text{ N.m}$$

(ii) Kinetic energy of the brake drum = Work done against friction or $\frac{1}{2} mv^2 = T_B.\omega$

or $\frac{160}{2} \times \left(\frac{\pi \times 0.44 \times 1500}{60} \right)^2 = 86.8 \times 2\pi n$

or $80 \times 1194.2 = 545.4\, n$ or $n = 175$

Time taken, $t = \frac{n}{N} = \frac{175}{1500/60} = 7$ s

Example 15.4 *A spring-operated pivoted shoe brake shown in Fig. 15.4 (a) is used for a wheel diameter of 500 mm. The angle of contact is 90° and the coefficient of friction is 0.3. The force applied by the spring on each arm is 5 kN. Determine the brake torque on the wheel.*

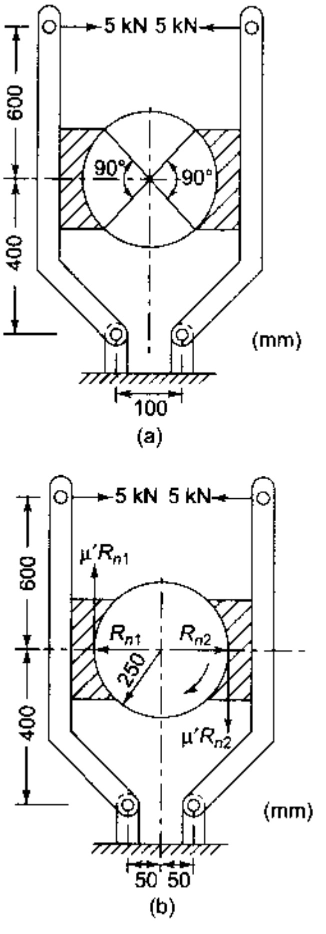


Fig. 15.4

Solution

$\mu = 0.3$;

$F = 5000$ N;

$d = 500$ mm,

$r = 250$ mm

Assuming the rotation to be clockwise, the various forces acting on the two blocks are shown in Fig. 15.4(b).

Now, $\mu' = \mu \left(\frac{4 \sin(\theta/2)}{\theta + \sin\theta} \right)$

$= 0.3 \left(\frac{4 \sin 45°}{(\pi/2) + \sin 90°} \right) = 0.33$

For the left-hand side block, taking moments about O_1,

$F \times 1 - R_{n1} \times 0.4 + \mu' R_{n1} \times (0.25 - 0.05) = 0$

$5000 \times 1 - R_{n1} \times 0.4 + 0.33 \times R_{n1} \times 0.2 = 0$

$R_{n1} = 14\,970$ N

For the right-hand side block, taking moments about O_2,

$5000 \times 1 - R_{n2} \times 0.4 - 0.33 \times R_{n2} \times 0.2 = 0$

$R_{n2} = 10\,730$ N

Maximum braking torque, $T_B = \mu' (R_{n1} + R_{n2})\, r$

$= 0.33\,(14\,970 + 10\,730) \times 0.25$

$= 2120$ N.m

Example 15.5 *Figure 15.5(a) shows an arrangement of a double block shoe brake. The force to each block is applied by means of a turn buckle with right and left-handed threads of six-start with a lead of 40 mm. The diameter of the turn buckle is 20 mm and it is rotated by a lever. The angle subtended by each block is 80°. The coefficient of friction for the brake blocks is 0.3 and for the screw and the nut, 0.18. Determine the brake torque applied by a force of 80 N at the end of the lever.*

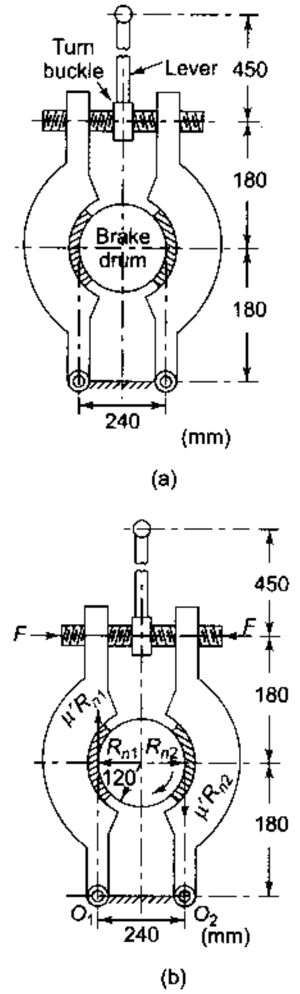


Fig. 15.5

Solution For screw and nut: lead = 40 mm, d = 20 mm, $\mu = 0.18$

For brake blocks, $\mu = 0.3$; For lever: $l = 450$ mm, $F' = 80$ N

Diameter of the brake drum = distance between the pivot = 240 mm

Assume the rotation of the drum to be clockwise. The various forces on the two blocks are shown in Fig. 15.5(b).

For the screw and nut,

$$\tan\alpha = \frac{\text{lead}}{\pi d} = \frac{40}{\pi \times 20} = 0.637$$

or $\alpha = 32.5°$

$\mu = 0.18$ or $\tan\phi = 0.18$ or $\phi = 10.2°$

Torque shared by each side of the spindle

$$= \frac{F' \times l}{2} = \frac{80 \times 450}{2} = 18\,000 \text{ N.mm}$$

If F be the force applied on each block along the screw axis,

$T = F\tan(\alpha + \varphi).r$ or 18 000

$= F\tan(32.5° + 10.2°) \times (20/2)$ or $F = 1951$ N

$$\mu' = \mu\left(\frac{4\sin(\theta/2)}{\theta + \sin\theta}\right)$$

$$= 0.3\left(\frac{4\sin 40°}{(80\pi/180) + \sin 80°}\right) = 0.324$$

For the left-hand side block, taking moments about O_1,

$F \times 0.36 - R_{n1} \times 0.18 = 0$

or $1951 \times 0.36 - R_{n1} \times 0.18 = 0$

or $R_{n1} = 3902$ N

For the right-hand side block, taking moments about O_2,

$F \times 0.36 - R_{n2} \times 0.18 = 0$ or $R_{n2} = R_{n1} = 3902$ N

Maximum braking torque, $T_B = \mu'(R_{n1} + R_{n2})\,r$ $= 0.324\,(2 \times 3902) \times 0.12 = 303.4$ N.m

Example 15.6 *A double-block brake is operated by a sprocket-and-chain mechanism as shown in Fig. 15.6. As a force F is applied at the end of the lever, the sprocket causes tensions in the chains. The brake drum diameter is 240 mm. The angle of contact of each block is 90°. Determine the force F required to apply the brake if a power of 1.6 kW at 300 rpm is being transmitted by the system.*

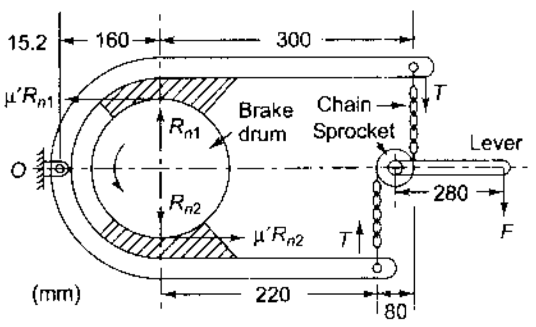


Fig. 15.6

Solution

$P = 1.6$ kW, $d = 240$ mm, $N = 300$ rpm, $\theta = 90°$

As the angle of contact is more than 40°,

$$\mu' = \mu\left(\frac{4\sin(\theta/2)}{\theta + \sin\theta}\right)$$

$$= 0.32\left(\frac{4\sin 45°}{90 \times \frac{\pi}{180} + \sin 90°}\right) = 0.385$$

Let T be the tension in the chains. Take moments about the centre of sprocket,

$$F \times 280 = T \times 40 + T \times 40 \quad \text{or} \quad T = 3.5\,F \qquad \text{(i)}$$

Upper shoe block: Let T be the tension in the chain. Taking moments about the fulcrum O,

$$T \times (160 + 300) - R_{n1} \times 160 - 0.385\,R_{n1} \times 120 = 0$$

or $460 \times 3.5\,F = 206.2\,R_{n1}$ or $R_{n1} = 7.808\,F$

Lower shoe block: Taking moments about the fulcrum O

$$T \times (160 + 220) - R_{n2} \times 160 + 0.385\,R_{n2} \times 120 = 0$$

or $380 \times 3.5\,F = 113.8\,R_{n2}$ or $R_{n2} = 11.69\,F$

or Maximum braking torque, $T_B = \mu'(R_{n1} + R_{n2})\,r$

$= 0.385\,(7.812\,F + 11.69\,F) \times 0.12$

$= 0.9$ F

As $P = T_B \times \omega$

$$1600 = 0.9\,F \times \frac{2\pi \times 300}{60}$$

$F = \underline{56.6\text{ N}}$

15.3 BAND BRAKE

It consists of a rope, belt or flexible steel band (lined with friction material) which is pressed against the external surface of a cylindrical drum when the brake is applied. The force is applied at the free end of a lever (Fig. 15.7).

Brake torque on the drum = $(T_1 - T_2)\,r$

where r is the effective radius of the drum.

The ratio of the tight and the slack side tensions is given by $T_1/T_2 = e^{\mu\theta}$ on the assumption that the band is on the point of slipping on the drum.

The effectiveness of the force F depends upon the

- direction of rotation of the drum
- ratio of lengths a and b
- direction of the applied force F

To apply the brake to the rotating drum, the band has to be tightened on the drum. This is possible if

(i) F is applied in the downward direction when $a > b$

(ii) F is applied in the upward direction when $a < b$

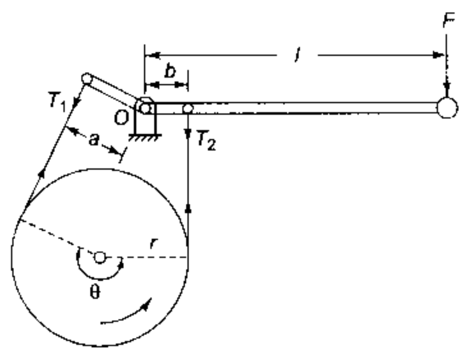


Fig. 15.7

If the force applied is not as above, the band is further loosened on the drum which means no braking effect is possible.

(i) $a > b$, F Downwards

(a) Rotation Counter-clockwise For counter-clockwise rotation of the drum, the tight and the slack sides of the band will be as shown in Fig. 15.7.

Considering the forces acting on the lever and taking moments about the pivot,

$$F\,l - T_1\,a + T_2\,b = 0$$

or

$$F = \frac{T_1 a - T_2 b}{l} \qquad (15.4)$$

As $T_1 > T_2$ and $a > b$ under all conditions, the effectiveness of the brake will depend upon the force F.

(b) Rotation Clockwise In this case, the tight and the slack sides are reversed as shown in Fig. 15.8

Now, $F\,l - T_2\,a + T_1\,b = 0$ or $F = \dfrac{T_2 a - T_1 b}{l}$

As $T_2 < T_1$ and $a > b$, the brake will be effective as long as $T_2.a$ is greater than $T_1.b$

or $T_2\,a > T_1\,b$ or $\dfrac{T_2}{T_1} > \dfrac{b}{a}$

i.e., as long as the ratio of T_2 to T_1 is greater than the ratio b/a.

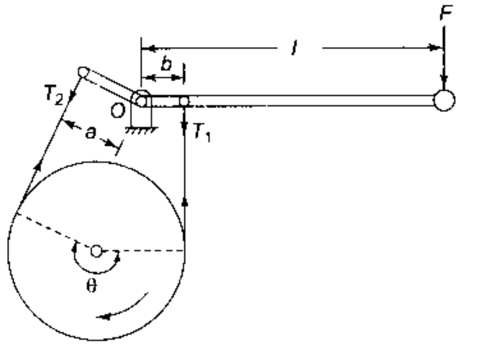


Fig. 15.8

When $\dfrac{T_2}{T_1} \le \dfrac{a}{b}$, F is zero or negative, i.e., the brake becomes self-locking as no force is needed to apply the brake. Once the brake has been engaged, no further force is required to stop the rotation of the drum.

(ii) $a < b$, F upwards

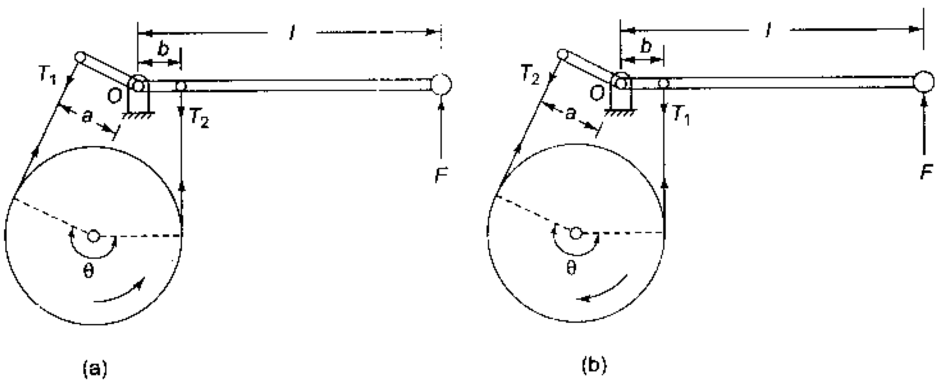


Fig. 15.9

(a) Rotation Counter-clockwise The tight and the slack sides will be as shown in Fig. 15.9(a). Therefore,

$$F\,l + T_1\,a - T_2\,b = 0$$

or
$$F = \frac{T_2 b - T_1 a}{l} \tag{15.5}$$

As $T_2 < T_1$ and $b > a$, the brake is operative only as long as

$$T_2\,b > T_1\,a \quad \text{or} \quad \frac{T_2}{T_1} > \frac{a}{b}$$

Once T_2/T_1 becomes equal to a/b, F required is zero and the brake becomes self-locking.

(b) Rotation Clockwise The tight and the slack sides are shown in Fig. 15.9(a).

From Fig. 15.9(b), $F\,l - T_1\,b + T_2\,a = 0 \quad \text{or} \quad F = \frac{T_1 b - T_2 a}{l}$

As $T_1 > T_2$ and $b > a$, under all conditions, the effectiveness of the brake will depend upon the force F.

- When $a = b$, the band cannot be tightened and thus, the brake cannot be applied.
- The band brake just discussed is known as a *differential band brake*. However, if either a or b is made zero, a *simple band brake* is obtained. If $b = 0$ (Fig.15.10) and F downwards,

$$F.l - T_1\,a = 0$$

or
$$F = T_1 \frac{a}{l} \tag{15.6}$$

Similarly, the force can be found for the other cases.
Note that such a brake can neither have self-energising properties nor it can be self-locked.

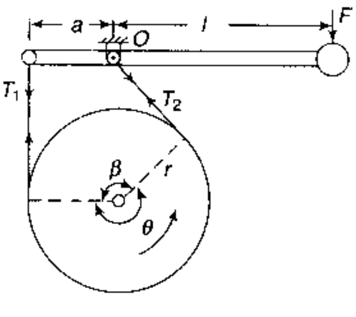


Fig. 15.10

- The brake is said to be more effective when maximum braking force is applied with the least effort F.
For case (i), when $a > b$ and F is downwards, the force (effort) F required is less when the rotation is clockwise assuming that the brake is effective.
For case (ii), when $a < b$ and F is upwards, F required is less when the rotation is counter-clockwise assuming that the brake is effective.
Thus, for the given arrangement of the differential brake, it is more effective when
(a) $a > b$, F downwards, rotation clockwise
(b) $a > b$, F upwards, rotation counter-clockwise
- The advantage of self-locking is taken in hoists and conveyers where motion is permissible in only one direction. If the motion gets reversed somehow, the self-locking is engaged which can be released only by reversing the applied force.
- It is seen in (v) that a differential band brake is more effective only in one direction of rotation of the drum. However, a two-way band brake can also be designed which is equally effective for both the directions of rotation of the drum (Fig. 15.11). In such a brake, the two lever arms are made equal.

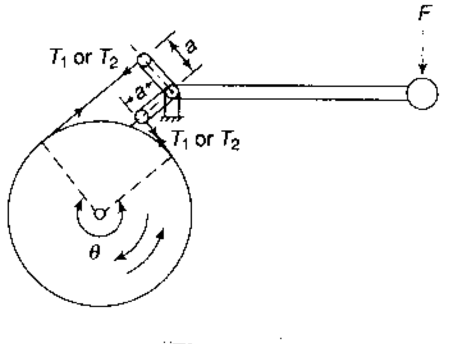


Fig. 15.11

For both directions of rotation of the drum,

$$Fl - T_1 a - T_2 a = 0$$

or

$$F = (T_1 + T_2)\frac{a}{l} \quad (15.7)$$

Example 15.7 *A differential band brake has a drum with a diameter of 800 mm. The two ends of the band are fixed to the pins on the opposite sides of the fulcrum of the lever at distances of 40 mm and 200 mm from the fulcrum. The angle of contact is 270° and the coefficient of friction is 0.2. Determine the brake torque when a force of 600 N is applied to the lever at a distance of 800 mm from the fulcrum.*

Solution $F = 600$ N, $l = 800$ mm, $r = 400$ mm, $\theta = 270°$ and $\mu = 0.2$

- Assuming $a = 200$ mm and $b = 40$ mm, i.e., $a > b$, F must act downwards to apply the brake (Fig. 15.7).

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.2 \times 270 \times \frac{\pi}{180}} = 2.57$$

Counter-clockwise rotation of the drum

Taking moments about the fulcrum,

$Fl - T_1\, a + T_2\, b = 0$

$600 \times 800 - 2.57\, T_2 \times 200 + T_2 \times 40 = 0$

or $T_2 = 1012.7$ N and

$T_1 = 1012.7 \times 2.57 = 2602.5$ N

Braking torque, $T_B = (2602.5 - 1012.7) \times 0.4$
$= 636$ N.m

Clockwise rotation of the drum

Taking moments about the fulcrum O,

$Fl + T_1\, b - T_2\, a = 0$ (Fig. 15.8)

$600 \times 800 + 2.57\, T_2 \times 40 - T_2 \times 200 = 0$

or $600 \times 800 = T_2\,(200 - 2.57 \times 40)$

or $T_2 = 4938$ N and $T_1 = 4938 \times 2.57 = 12\,691$ N

$T_B = (T_1 - T_2)\, r = (12\,691 - 4938) \times 0.4$
$= 3101$ N.m

- Assuming $a = 40$ mm and $b = 200$ mm, i.e., $a < b$, F must act upwards to apply the brake.

Counter-clockwise rotation of the drum

$600 \times 800 + 2.57\, T_2 \times 40 - T_2 \times 200 = 0$ (Fig. 15.9a)

or $600 \times 800 = T_2\,(200 - 2.57 \times 40)$

or $T_2 = 4938$ N and $T_1 = 4938 \times 2.57 = 12\,691$ N

Braking torque, $T_B = (T_1 - T_2)\, r$
$= (12\,691 - 4938) \times 0.4 = 3101$ N.m

Clockwise rotation of the drum

$600 \times 800 + T_2 \times 40 - 2.57\, T_2 \times 200 = 0$ (Fig. 15.9b)

or $T_2 = 1012.7$ N

and $T_1 = 1012.7 \times 2.57 = 2602.5$ N

$T_B = (2602.5 - 1012.7) \times 0.4 = \underline{636\text{ N.m}}$

The above results show that the effectiveness of the brake in one direction of rotation is equal to the effectiveness in the other direction if the distances of the pins on the opposite sides of the fulcrum are changed and the force is applied in the proper direction so that the band is tightened.

Example 15.8 *A simple band brake (Fig. 15.12) is applied to a shaft carrying a flywheel of 250-kg mass and of radius of gyration of 300 mm. The shaft speed is 200 rpm. The drum diameter is 200 mm and the coefficient of friction is 0.25. Determine the*

(i) brake torque when a force of 120 N is applied at the lever end

(ii) number of turns of the flywheel before it comes to rest

(iii) time taken by the flywheel to come to rest

Solution

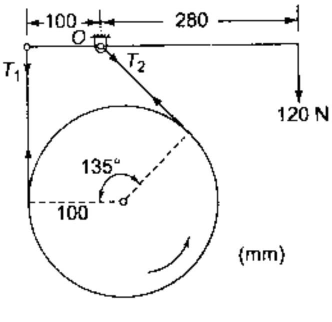


Fig. 15.12

$m = 250$ kg $\quad \mu = 0.25$
$k = 300$ mm $\quad r = 100$ mm
$N = 200$ rpm $\quad a = 100$ mm
$\beta = 135°$ $\quad l = 280$ mm

(i) $\theta = 360° - 135° = 225°$

or $\quad \theta = 225 \times \frac{\pi}{180} = 3.93$ rad

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.25 \times 3.93} = 2.67$$

Taking moments about O,

$F \times l - T_1 \times a = 0$

$120 \times 280 - T_1 \times 100 = 0$

$T_1 = 336$ N

$$T_2 = \frac{336}{2.67} = 125.8 \text{ N}$$

$T_B = (336 - 125.8) \times 0.1 = \underline{21 \text{ N.m}}$

(ii) KE of the flywheel

$$= \frac{1}{2} I\omega^2 = \frac{1}{2} mk^2 \left(\frac{2\pi N}{60}\right)^2$$

$$= \frac{1}{2} \times 250 \times (0.3)^2 \times \left(\frac{2\pi \times 200}{60}\right)^2$$

$= 4935$ N.m

Let the KE be used to overcome the work done by the braking torque in n revolutions. Then

$T_B \times$ Angular displacement $= KE$ of flywheel

$21 \times 2\pi n = 4935$

$n = 37.4$ revolutions

(iii) For uniform retardation, average speed $= 200/2 = 100$ rpm

$$\text{Time taken} = \frac{n}{N} = \frac{37.4}{100} \text{ min}.$$

$$= \frac{37.4}{100/60} = \underline{22.44 \text{ s}}.$$

Example 15.9 *A simple band brake is applied to a drum of 560-mm diameter which rotates at 240 rpm. The angle of contact of the band is 270°. One end of the band is fastened to a fixed pin and the other end to the brake lever, 140 mm from the fixed pin. The brake lever is 800 mm long and is placed perpendicular to the diameter that bisects the angle of contact. Assuming the coefficient of friction as 0.3, determine the necessary pull at the end of the lever to stop the drum if 40 kW of power is being absorbed. Also, find the width of the band if its thickness is 3 mm and the maximum tensile stress is limited to 40 N/mm².*

Solution The brake is shown in Fig. 15.13.

$N = 240$ rpm, $d = 560$ mm, $r = 280$ mm, $\theta = 270°$, $a = 140$ mm, $l = 800$ mm, $\mu = 0.3$, $P = 40$ kW, $t = 3$ mm, $\sigma = 40$ N/mm².

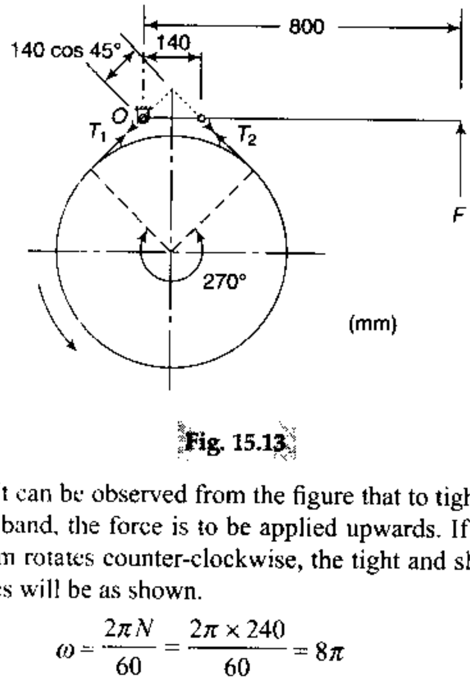


Fig. 15.13

It can be observed from the figure that to tighten the band, the force is to be applied upwards. If the drum rotates counter-clockwise, the tight and slack sides will be as shown.

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 8\pi$$

$$\text{Angle of lap, } \theta = 270 \times \frac{\pi}{180} = 1.5\pi \text{ rad}$$

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 1.5\pi} = 4.11$$

Also, $P = T_B \cdot \omega$

$= [(T_1 - T_2)r] \cdot \omega$

or $\quad 40\,000 = (T_1 - T_2) \times 0.28 \times 8\pi$

$(T_1 - T_2) = 5684$

or $\quad 4.11\, T_2 - T_2 = 5684$

$T_2 = 1828$ N

$T_1 = 1828 \times 4.11 = 7514$ N

Take moments of the forces on the lever about the fulcrum O,

$F \times 800 = 1828 \times 140 \cos 45°$

$F = \underline{226.2 \text{ N}}$

Let b be the width of the band.

Maximum tension, $T_1 = \sigma . b . t$

or $7514 = 40 \times b \times 3$

or $b = \underline{62.6 \text{ mm}}$

Observe that if the drum rotates clockwise, the brake is less effective as in that case tight and slack sides are interchanged and the force required to apply the same brake torque is more which is

$F \times 800 = 7515 \times 140 \cos 45°$

$F = 930$ N

Example 15.10 *A crane is required to support a load of 1.2 tonnes on the rope round its barrel of 400 mm diameter (Fig.15.14). The brake drum which is keyed to the same shaft as the barrel has a diameter of 600 mm. The angle of contact of the band brake is 275° and the coefficient of friction is 0.22. Determine the force required at the end of the lever to support the load. Take a = 150 mm and l = 750 mm.*

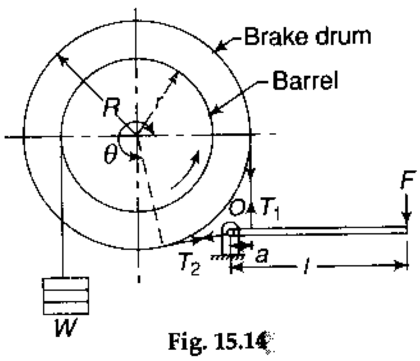


Fig. 15.14

Solution

$W = (1.2 \times 1000 \times 9.81)$ N

$R = 300$ mm

$r = 200$ mm

$\mu = 0.22$

$\theta = 275°$

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.22 \times 275 \times \frac{\pi}{180}}$$

$= 2.87$

For equilibrium,

$(T_1 - T_2)\, R = W \times r$

or $(2.87\, T_2 - T_2) \times 300 = (1.2 \times 1000 \times 9.81) \times 200$

$T_2 = 4197$ N.m

$T_1 = 4197 \times 2.87 = \underline{12\,045 \text{ N.m}}$

Taking moments about O,

$F \times l - T_1 \times a = 0$

or $F \times 750 - 12\,045 \times 150 = 0$

or $F = \underline{2407 \text{ N}}$

15.4 BAND AND BLOCK BRAKE

A band and block brake consists of a number of wooden blocks secured inside a flexible steel band. When the brake is applied, the blocks are pressed against the drum. Two sides of the band become tight and slack as usual. Wooden blocks have a higher coefficient of friction, Thus, increasing the effectiveness of the brake. Also, such blocks can be easily replaced on being worn out [Fig.15.15(a)].

Each block subtends a small angle of 2θ at the centre of the drum. The frictional force on the blocks acts in the direction of rotation of the drum. For n blocks on the brake,

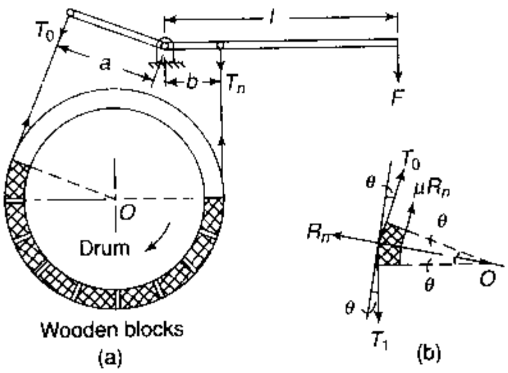


Fig. 15.15

Let T_0 = tension on the slack side
T_1 = tension on the tight side after one block
T_2 = tension on the tight side after two blocks
....
....
T_n = tension on the tight side after n blocks
μ = coefficient of friction
R_n = normal reaction on the block

The forces on one block of the brake are shown in Fig.15.15(b).

For equilibrium,

$$(T_1 - T_0)\cos\theta = \mu R_n$$

$$(T_1 + T_0)\sin\theta = R_n$$

or
$$\frac{T_1 - T_0}{T_1 + T_0}\frac{1}{\tan\theta} = \mu$$

or
$$\frac{T_1 - T_0}{T_1 + T_0} = \frac{\mu\tan\theta}{1}$$

or
$$\frac{(T_1 - T_0) + (T_1 + T_0)}{(T_1 - T_0) - (T_1 + T_0)} = \frac{\mu\tan\theta + 1}{\mu\tan\theta - 1}$$

$$-\frac{2T_1}{2T_0} = -\frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}$$

$$\frac{T_1}{T_0} = \frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}$$

Similarly,
$$\frac{T_2}{T_1} = \frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}\text{, and so on.}$$

....
....

and
$$\frac{T_n}{T_n - 1} = \frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}$$

$$\frac{T_n}{T_0} = \frac{T_n}{T_{n-1}}\frac{T_{n-1}}{T_{n-2}}\cdots\frac{T_2}{T_1}\frac{T_1}{T_0}$$

$$= \left(\frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}\right)^n \tag{15.8}$$

Example 15.11 *A band and block brake has 14 blocks. Each block subtends an angle of 14° at the centre of the rotating drum. The diameter of the drum is 750 mm and the thickness of the blocks is 65 mm. The two ends of the band are fixed to the pins on the lever at distances of 50 mm and 210 mm from the fulcrum on the opposite sides. Determine the least force required to be applied at the lever* *at a distance of 600 mm from the fulcrum if the power absorbed by the blocks is 180 kW at 175 rpm. Coefficient of friction between the blocks and the drum is 0.35.*

Solution

$N = 175$ rpm, $d = 750$ mm, $\theta = 7°$, $\mu = 0.35$, $P = 180$ kW, $t = 65$ mm, $l = 600$ mm

Refer Fig. 15.15.

$$P = (T_{14} - T_0).v = (T_{14} - T_0).\frac{\pi DN}{60}$$

$$\therefore 180\,000 = (T_{14} - T_0) \times \frac{\pi \times (0.75 + 2 \times 0.065) \times 175}{60}$$

or $T_{14} - T_0 = 22\,323$ N

$$\frac{T_{14}}{T_0} = \left(\frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}\right)^n = \left(\frac{1 + 0.35 \tan 7°}{1 - 0.35 \tan 7°}\right)^{14} = 3.334$$

or $2.334\, T_0 = 22\,323$ or $T_0 = 9564$ N

and $T_{14} = 22\,323 + 9564 = 31\,887$ N

Assume $a = 210$ mm and $b = 50$ mm (Fig. 15.15)

As $a > b$, F must be downwards and rotation clockwise for maximum braking torque. Taking moments about the fulcrum,

$$F \times l - T_0\, a + T_{12}\, b = 0$$

$$F \times 600 - 9564 \times 210 + 31887 \times 50 = 0$$

$$600\, F = 414\,090 \quad \text{or} \quad F = 690 \text{ N}$$

Example 15.12 *A band and block brake having 12 blocks, each of which subtends an angle of 16° at the centre, is applied to a rotating drum with a diameter of 600 mm. The blocks are 75 mm thick. The drum and the flywheel mounted on the same shaft have a mass of 1800 kg and have a combined radius of gyration of 600 mm. The two ends of the band are attached to pins on the opposite sides of the brake fulcrum at distances of 40 mm and 150 mm from it. If a force of 250 N is applied on the lever at a distance of 900 mm from the fulcrum, find the*

(i) maximum braking torque

(ii) angular retardation of the drum

(iii) time taken by the system to be stationary from the rated speed of 300 rpm.

Take coefficient of friction between the blocks and the drum as 0.3.

Solution

$F = 250$ N, $d = 600$ mm, $\theta = 8°$, $t = 75$ mm, $l = 900$ mm, $k = 600$ mm, $m = 1800$ kg, $n = 12$, $N = 300$ rpm, $\mu = 0.3$

Refer Fig. 15.15.

(i) $$\frac{T_{12}}{T_0} = \left(\frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}\right)^n$$

$$= \left(\frac{1 + 0.3 \tan 8°}{1 - 0.3 \tan 8°}\right)^{12} = 2.752$$

Assume $a = 150$ mm and $b = 40$ mm

As $a > b$, F must be downwards and the rotation is clockwise for maximum braking torque. Taking moments about the fulcrum,

$$F \times l - T_0\, a - T_{12}\, b = 0$$

$$250 \times 900 - T_0 \times 150 + 2.752\, T_0 \times 40 = 0$$

$$T_0\,(150 - 2.752 \times 40) = 250 \times 900$$

$$T_0 = 5636 \text{ N}$$

$$T_{12} = 5636 \times 2.752 = 15\,511 \text{ N}$$

Maximum braking torque, $T_B = (T_{12} - T_0) \times \dfrac{d}{2}$

$$= (15\,511 - 5636) \times \left(\frac{0.6 + 0.075 \times 2}{2}\right)$$

$= \underline{3703 \text{ N.m}}$

(ii) $T_B = I\alpha = mk^2\alpha$

$$3703 = 1800 \times (0.6)^2 \times \alpha$$

$\alpha = \underline{5.71 \text{ rad/s}^2}$

(iii) Initial angular speed,

$$\omega_0 = \frac{2\pi \times 300}{60} = 31.4 \text{ rad/s}$$

Final angular speed, $\omega = 0$

$\therefore\ \omega = \omega_0 - \alpha t$ (α negative due to retardation)

or $0 = 31.4 - 5.71\, t$

$t = \underline{5.5 \text{ s}}$

15.5 INTERNAL EXPANDING SHOE BRAKE

Earlier, automobiles used band brakes which were exposed to dirt and water. Their heat dissipation capacity was also poor. These days, band brakes have been replaced by internal expanding shoe brakes having at least one self-energising shoe per wheel. This results in tremendous friction, giving great braking power without excessive use of pedal pressure.

Figure 15.16 shows an internal shoe automobile brake. It consists of two semi-circular shoes which are lined with a friction material such as *ferodo*. The shoes press against the inner flange of the drum when the brakes are applied. Under normal running of the vehicle, the drum rotates freely as the outer diameter of the shoes is a little less than the internal diameter of the drum.

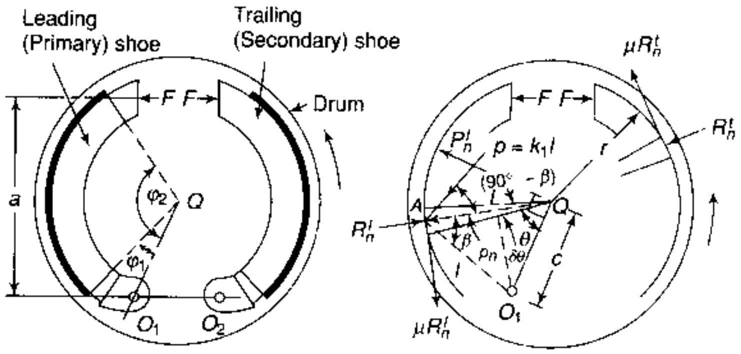
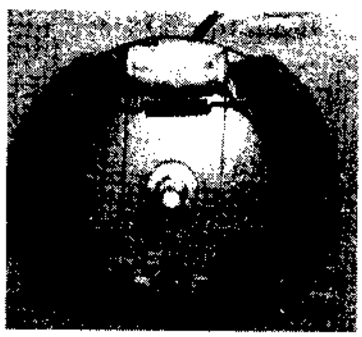


Fig. 15.16



Internal expanding shoe brake mechanism (without brake drum).

The actuating force F is usually applied by two equal-diameter pistons in a common hydraulic cylinder and is applied equally in magnitude to each shoe. For the shown direction of the drum rotation, the left shoe is known as the *leading* or *forward shoe* and the right as the *trailing* or *rear shoe*.

Assuming that each shoe is rigid as compared to the friction surface, the pressure p at any point A on the contact surface of the shoe will be proportional to its distance l from the pivot.

Considering the leading shoe,

$p \propto l = k_1 l$ where k_1 is a constant.

The direction of p is perpendicular to OA.

The normal pressure, $p_n = k_1 l \cos(90° - \beta) = k_1 l \sin\beta$

$= k_1 c \sin\theta$ $\quad$ $(O_1 L = l\sin\beta = c\sin\theta)$

$= k_2 \sin\theta$ $\quad$ where k_2 is another constant

p_n is maximum when $\theta = 90°$

Let P_n^l = maximum intensity of normal pressure on the leading shoe.

$$p_{n\,\max} = P_n^l = k_2 \sin 90° = k_2$$

or

$$p_n = P_n^l \sin\theta \qquad (15.9)$$

Let w = width of brake lining

μ = coefficient of friction

Consider a small element of brake lining on the leading shoe that makes an angle $\delta\theta$ at the centre.

Normal reaction on the differential surface,

$$R_n^l = \text{Area} \times \text{Pressure}$$

$$= (r\,\delta\theta\,w)\,p_n$$
$$= r\,\delta\theta\,w\,P^l_n \sin\theta$$

Taking moments about the fulcrum O_1,

$$Fa - \sum_{\varphi 1}^{\varphi 2} R^l_n c \sin\theta + \sum_{\varphi 1}^{\varphi 2} \mu R^l_n (r - c\cos\theta) = 0 \tag{15.10}$$

where

$$\sum_{\varphi 1}^{\varphi 2} R^l_n c \sin\theta = \int_{\varphi 1}^{\varphi 2} rcwP^l_n \sin^2\theta\, d\theta$$

$$= \int_{\varphi 1}^{\varphi 2} rcwP^l_n \frac{1}{2}(1 - \cos 2\theta)\, d\theta$$

$$= rcwP^l_n \frac{1}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)_{\varphi 1}^{\varphi 2}$$

$$= \frac{rcwP^l_n}{4}(2\varphi_2 - 2\varphi_1 - \sin 2\varphi_2 + \sin 2\varphi_1)$$

and

$$\sum_{\varphi 1}^{\varphi 2} \mu R^l_n (r - c\cos\theta) = \int_{\varphi 1}^{\varphi 2} \mu r^2 wP^l_n \sin\theta\, d\theta - \int_{\varphi 1}^{\varphi 2} \mu rcwP^l_n \sin\theta\cos\theta\, d\theta$$

$$= \mu r^2 wP^l_n (-\cos\theta)_{\varphi 1}^{\varphi 2} - \int_{\varphi 1}^{\varphi 2} \mu rcwP^l_n \frac{1}{2}\sin 2\theta\, d\theta$$

$$= \mu r^2 wP^l_n (\cos\varphi_1 - \cos\varphi_2) - \mu rcwP^l_n \frac{1}{2}\left(\frac{-\cos 2\theta}{2}\right)_{\varphi 1}^{\varphi 2}$$

$$= \frac{\mu rwP^l_n}{4}[4r(\cos\varphi_1 - \cos\varphi_2) - c(\cos 2\varphi_1 - \cos 2\varphi_2)]$$

Taking moments about the fulcrum O_2 for the trailing shoe,

$$Fa - \sum_{\varphi 1}^{\varphi 2} R^t_n c \sin\theta - \sum_{\varphi 1}^{\varphi 2} \mu R^t_n (r - c\cos\theta) = 0$$

where

$$\sum_{\varphi_1}^{\varphi_2} R^t_n c \sin\theta = \frac{rcwP^t_n}{4}[2\varphi_2 - 2\varphi_1 - \sin 2\varphi_2 + \sin 2\varphi_1]$$

and

$$\sum_{\varphi_1}^{\varphi_2} \mu R^t_n (r - c\cos\theta) = \frac{\mu rwP^t_n}{4}[4r(\cos\varphi_1 - \cos\varphi_2) - c(\cos 2\varphi_1 - \cos 2\varphi_2)]$$

Thus P^l_n and P^t_n, the maximum pressure intensities on the leading and the trailing shoes, can be determined.

Braking torque,

$$T_B = \sum_{\varphi_1}^{\varphi_2} \mu R^l_n r + \sum_{\varphi_1}^{\varphi_2} \mu R^t_n r$$

$$= \int_{\varphi_1}^{\varphi_2} \mu r^2 wP^l_n \sin\theta\, d\theta + \int_{\varphi_1}^{\varphi_2} \mu r^2 wP^t_n \sin\theta\, d\theta$$

$$= r^2 \mu w(P_n^l + P_n^t)(-\cos\varphi)_{\varphi_1}^{\varphi_2}$$

$$= r^2 \mu w(P_n^l + P_n^t)(\cos\varphi_1 - \cos\varphi_2) \tag{15.11}$$

Note that for the same applied force F on each shoe, P^l_n is not equal to P^t_n and $P^l_n > P^t_n$. Usually, more than 50% of the total braking torque is supplied by the leading shoe.

Also note that the leading shoe is self-energizing whereas the trailing shoe is not. This is because the friction forces acting on the leading shoe help the applied force F, and that on the trailing shoe oppose it. On reversing the direction of drum rotation, the right shoe will become self-energizing whereas the left will not be so any longer.

In Eq. 15.10, if the third term exceeds the second term on the LHS, F will be negative and the brake becomes self-locking. A brake should be self-energizing but not self-locking. The amount of self-energizing is measured by the ratio of the friction moment and the normal reaction moment, i.e., the ratio of the third term to the second term. When this ratio is equal to or more than unity, the brake is self-locking. When the ratio is less than unity (more than zero), the brake is self-energizing.

Example 15.13 *The following data refer to an internal expanding shoe brake shown in Fig. 15.16:*

Force F on each shoe	$= 180$ N
Coefficient of friction, μ	$= 0.3$
Internal radius of the brake drum, r	$= 150$ mm
Width of the brake lining, w	$= 40$ mm
Distance: a = 200 mm	$c = 120$ mm
Angles: $\varphi_1 = 30°$	$\varphi_2 = 135°$

Determine the bracking torque applied when the drum rotates (i) counter-clockwise, and (ii) clockwise.

Solution

(i) Rotation counter-clockwise

For the leading shoe

$$Fa - \int_{\varphi_1}^{\varphi_2} R_n^l c \sin\theta + \int_{\varphi_1}^{\varphi_2} \mu R_n^l (r - c\cos\theta) = 0$$

$$180 \times 0.2 - \frac{0.15 \times 0.12 \times 0.04 \times P_n^l}{4} \left(2 \times 135 \times \frac{\pi}{180} - 2 \times 30 \times \frac{\pi}{180} - \sin 270° + \sin 60°\right)$$

$$+ \frac{0.3 \times 0.15 \times 0.04 \times P_n^l}{4} \begin{bmatrix} 4 \times 0.15(\cos 30° - \cos 135°) \\ -0.12(\cos 60° - \cos 270°) \end{bmatrix} = 0$$

$$36 - 0.000\,996 P_n^l + 0.000\,398 P_n^l = 0$$

$$P_n^l = 60\,201 \text{ N/m}^2$$

For the trailing shoe

$$36 - 0.000\,996 P_n^t - 0.000\,398 P_n^t = 0$$

$$P_n^t = 25\,825 \text{ N/m}^2$$

Braking torque,

$$T_B = r^2 \mu w(P_n^l + P_n^t)(\cos\varphi_1 - \cos\varphi_2)$$

$= (0.15)^2 \times 0.3 \times 0.04\ (60\,201 + 25\,825)(\cos 30° - \cos 135°)$

= <u>36.54 N.m</u>

(ii) Rotation clockwise

When the rotation is reversed, P^l_n and P^t_n are interchanged and Thus, the braking torque is the same.

15.6 EFFECT OF BRAKING

Consider a vehicle moving up an inclined plane (Fig. 15.17).

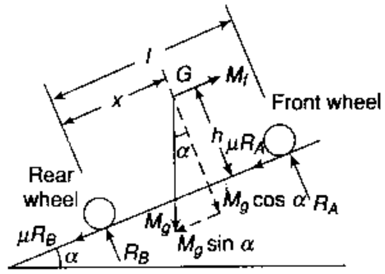


Fig. 15.17

Brakes applied to rear wheels only

Let M = mass of vehicle

α = angle of inclination of the plane with horizontal

R_A, R_B = reactions of the ground on the front and rear wheels respectively

f = retardation of the vehicle

l = wheel base of the car

h = height of centre of mass of the vehicle from the inclined surface

x = distance of the centre of mass from the rear axle

μ = coefficient of friction between the ground and the tyres

For equilibrium,

$$R_A + R_B = Mg\cos\alpha \tag{i}$$

$$\mu R_B + Mg\sin\alpha = Mf \tag{ii}$$

where f is the retardation of the vehicle.

Taking moments about G, the centre of mass of the vehicle,

$$R_B x + \mu R_B \times h - R_A(l - x) = 0 \tag{iii}$$

From (i),

$$R_A = Mg\cos\alpha - R_B$$

∴ (iii) becomes,

$$R_B x + \mu R_B \times h - (Mg\cos\alpha - R_B)(l - x) = 0$$

or

$$R_B(x + \mu h + l - x) = Mg\cos\alpha\,(l - x)$$

$$R_B = \frac{Mg\cos\alpha\,(l - x)}{l + \mu h}$$

or

and thus (ii) becomes,

$$\mu\frac{Mg\cos\alpha\,(l - x)}{l + \mu h} + Mg\sin\alpha = Mf$$

or

$$f = g\cos\alpha\left[\frac{\mu(l - x)}{l + \mu h} + \tan\alpha\right] \tag{15.12}$$

On a level road, $\alpha = 0$, and so

$$f = g\frac{\mu(l - x)}{l + \mu h} \tag{15.13}$$

When the vehicle moves down a plane,

$$f = g\cos\alpha\left[\frac{\mu(l - x)}{l + \mu h} - \tan\alpha\right] \tag{15.14}$$

Brakes applied to front wheels only

$$R_A + R_B = Mg \cos \alpha \qquad \text{(iv)}$$

$$\mu R_A + Mg \sin \alpha = Mf \qquad \text{(v)}$$

Taking moments about G,

$$R_B x + \mu R_A \times h - R_A (l - x) \qquad \text{(vi)}$$

From (iv) and (vi),

$$(Mg \cos \alpha - R_A) x + \mu R_A \times h - R_A (l - x) = 0$$

or

$$Mg x \cos \alpha = R_a (x - \mu h + l - x)$$

$$R_A = \frac{Mgx \cos \alpha}{l - \mu h}$$

Therefore (v) becomes, $\mu = \dfrac{Mgx \cos \alpha}{l - \mu h} + Mg \sin \alpha = Mf$

or

$$f = g \cos \alpha \left[\frac{\mu x}{l - \mu h} + \tan \alpha \right] \qquad (15.15)$$

On a level road, $\alpha = 0$, and therefore

$$f = g \frac{\mu x}{l - \mu h} \qquad (15.16)$$

On a down plane,

$$f = g \cos \alpha \left(\frac{\mu x}{l - \mu h} - \tan \alpha \right) \qquad (15.17)$$

Brakes applied to all four wheels

$$R_A + R_B = Mg \cos \alpha \qquad \text{(vii)}$$

$$\mu R_A + \mu R_B + Mg \sin \alpha = Mf \qquad \text{(viii)}$$

or

$$\mu (R_A + R_B) + Mg \sin \alpha = Mf$$

or

$$\mu Mg \cos \alpha + Mg \sin \alpha = Mf$$

or

$$f = g \cos \alpha (\mu + \tan \alpha) \qquad (15.18)$$

On a level road, $\alpha = 0$. Therefore

$$f = g \mu \qquad (15.19)$$

On a down plane,

$$f = g \cos \alpha (\mu - \tan \alpha) \qquad (15.20)$$

Example 15.14 *A vehicle having a wheel base of 3.2 m has its centre of mass at 1.4 m from the rear wheels and 55 mm from the ground level. It moves on a level ground at a speed of 54 km/h. Determine the distance moved by the car before coming to rest on applying the brakes to the*

(i) rear wheels
(ii) front wheels
(iii) all the four wheels

The coefficient of friction between the tyres and the road is 0.5.

Solution Let s be the distance moved by the car before coming to rest.

$$u = 54 \text{ km/h} = \frac{54\ 000}{3600} = 15 \text{ m/s}$$

(i) Brakes applied to rear wheels

$$f = g\frac{\mu(l-x)}{l+\mu h} = 9.81 \times \frac{0.5(3.2-1.4)}{3.2+0.5\times 0.55}$$

$$= 2.54 \text{ m/s}^2$$

If retardation is uniform, $v^2 - u^2 = -2fs$

$0 - u^2 = -2fs$

$$s = \frac{u^2}{2f} = \frac{15^2}{2\times 2.54} = \underline{44.3 \text{ m}}$$

(ii) Brakes applied to front wheels

$$f = g\frac{\mu x}{l-\mu h} = 9.81\times\frac{0.5\times 1.4}{3.2-0.5\times 0.55}$$

$$= 2.35 \text{ m/s}^2$$

$$s = \frac{u^2}{2f} = \frac{15^2}{2\times 2.35} = 47.9 \text{ m}$$

(iii) Brakes applied to all the four wheels

$f = gu = 9.81 \times 0.5 = 4.905 \text{ m/s}^2$

$$s = \frac{u^2}{2fs} = \frac{15^2}{2\times 4.90} = \underline{22.9 \text{ m}}$$

Example 15.15 ***A vehicle moves on a road that has a slope of 15°. The wheel base is 1.6 m and the centre of mass is at 0.72 m from the rear wheels and 0.8 m above the inclined plane. The speed of the vehicle is 45 km/h. The brakes are applied to all the four wheels and the coefficient of friction is 0.4. Determine the distance moved by the vehicle before coming to rest and the time taken to do so if it moves***

(i) up the plane

(ii) down the plane

Solution Let s be the distance moved by the car before coming to rest.

$$u = 45 \text{ km/h} = \frac{45\ 000}{3600} = 12.5 \text{ m/s}$$

(i) The vehicle moves up

$f = g\cos\alpha\,(\mu + \tan\alpha)$

$= 9.81 \times \cos 15° (0.4 + \tan 15°) = 6.33 \text{ m/s}^2$

If retardation is uniform, $v^2 - u^2 = -2fs$

$0 - u^2 = -2fs$

$$s = \frac{u^2}{2f} = \frac{12.5^2}{2\times 6.33} = \underline{12.34 \text{ m}}$$

Also, $v = u - ft$

or $0 = 12.5 - 6.33 \times t$

or $t = 1.97$ s

(ii) The vehicle moves down

$f = g\cos\alpha\,(\mu - \tan\alpha)$

$= 9.81 \times \cos 15° (0.4 - \tan 15°) = 1.25 \text{ m/s}^2$

$$s = \frac{u^2}{2f} = \frac{12.5^2}{2\times 1.25} = \underline{62.5 \text{ m}}$$

Also, $0 = 12.5 - 1.25 \times t$

or $t = \underline{10 \text{ s}}$

15.7 TYPES OF DYNAMOMETERS

There are mainly two types of dynamometers:

(i) ***Absorption Dynamometers*** In this type, the work done is converted into heat by friction while being measured. They can be used for the measurement of moderate powers only. Examples are prony brake dynamometer and rope brake dynamometer.

(ii) ***Transmission Dynamometers*** In this type, the work is not absorbed in the process, but is utilised after the measurement. Examples are the belt-transmission dynamometer and the torsion dynamometer.

15.8 PRONY BRAKE DYNAMOMETER

A prony brake dynamometer consists of two wooden blocks clamped together on a revolving pulley carrying a lever (Fig. 15.18). The friction between the blocks and the pulley tends to rotate the blocks in the direction of rotation of the shaft. However, the weight due to suspended mass at the end of the lever prevents this tendency. The grip of the blocks over the pulley is adjusted using the bolts of the clamp until the engine runs at the required speed. The mass added to the scale pan is such that the arm remains horizontal in the equilibrium position; the power of the engine is thus absorbed by the friction.

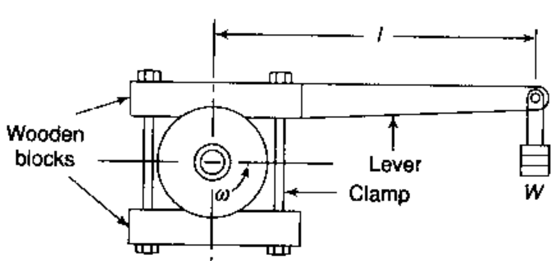


Fig. 15.18

Frictional torque = $Wl = Mgl$

$$\text{Power of the machine under test} = T\omega = Mgl\,\frac{2\pi N}{60}$$

$$= MNk$$

where k is a constant for a particular brake.

Note that the expression for power is independent of the size of the pulley and the coefficient of friction.

15.9 ROPE BRAKE DYNAMOMETER

In a rope brake dynamometer (Fig. 15.19), a rope is wrapped over the rim of a pulley keyed to the shaft of the engine. The diameter of the rope depends upon the power of the machine. The spacing of the ropes on the pulley is done by 3 to 4 U-shaped wooden blocks which also prevent the rope from slipping off the pulley. The upper end of the rope is attached to a spring balance whereas the lower end supports the weight of suspended mass.

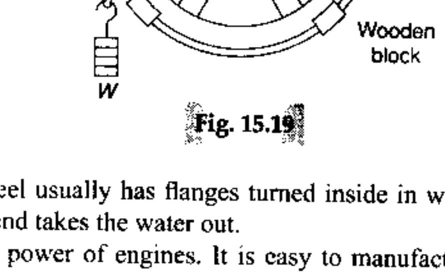


Fig. 15.19

$$\text{Power of the machine} = T\omega$$

$$= (F_t \times r)\,\omega$$

$$= (Mg - s)r\,\frac{2\pi N}{60}$$

If the power produced is high, so will be the heat produced due to friction between the rope and the wheel, and a cooling arrangement is necessary. For this, the channel of the flywheel usually has flanges turned inside in which water from a ripe is supplied. An outlet pipe with a flattened end takes the water out.

A rope brake dynamometer is frequently used to test the power of engines. It is easy to manufacture, inexpensive, and requires no lubrication.

If the rope is wrapped several times over the wheel, the tension on the slack side of the rope, i.e., the spring balance reading can be reduced to a negligible value as compared to the tension of the tight side (as $T_1/T_2 = e^{\mu\theta}$ and θ is increased). Thus, one can even do away with the spring balance.

Example 15.16 *The following data refer to a laboratory experiment with a rope brake:*

Diameter of the flywheel = 800 mm
Diameter of the rope = 8 mm
Dead weight on the brake = 40 kg
Speed of the engine = 150 rpm
Spring balance reading = 100 N
Find the power of the engine.

Solution

$$P = (Mg - s)r\frac{2\pi N}{60}$$

$$= (40 \times 9.81 - 100) \times (0.4 + 0.004)\frac{2\pi \times 150}{60}$$

$= 1855.6$ W

15.10 BELT TRANSMISSION DYNAMOMETER

The belt transmission dynamometer occupies a prominent position among transmission dynamometers. When a belt transmits power from one pulley to another, there exists a difference in tensions between the tight and slack sides. A dynamometer measures directly the difference in tensions ($T_1 - T_2$) while the belt is running.

Figure 15.20 shows a *Tatham* dynamometer. A continuous belt runs over the driving and the driven pulleys through two intermediate pulleys. The intermediate pulleys have their pins fixed to a lever with its fulcrum at the midpoint of the two pulley centres. As the lever is not pivoted at its midpoint, a mass at the left end is used for its initial equilibrium. When the belt transmits power, the lever tends to rotate in the counter-clockwise direction due to the difference of tensions on the tight and the slack sides. To maintain its horizontal position, a weight of the required amount is provided at the right end of the lever. Two stops, one on each side of the lever arm, are used to limit the motion of the lever.

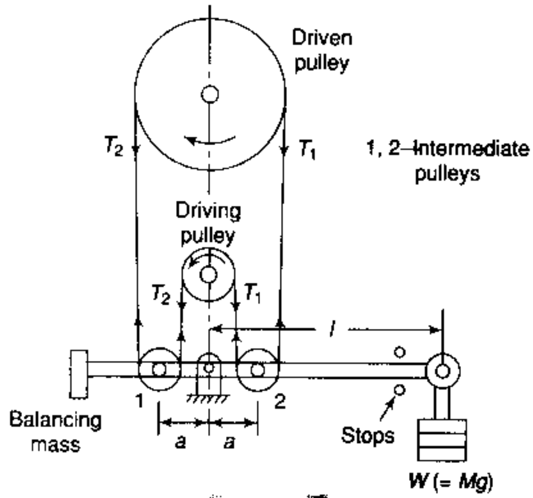


Fig. 15.20

Taking moments about the fulcrum,

$$Mgl - 2T_1a + 2T_2a = 0$$

$$Mgl - 2a(T_1 - T_2) = 0$$

$$T_1 - T_2 = \frac{Mgl}{2a}$$

Power, $P = (T_1 - T_2)v$

where v = belt speed in metres per second.

Example 15.17 *In a belt transmission dynamometer, the driving pulley rotates at 300 rpm. The distance between the centre of the driving pulley and the dead mass is 800 mm. The diameter of each of the driving as well as the intermediate pulleys is equal to 360 mm. Find the value of the dead mass required to maintain the lever in a horizontal position when the power transmitted is 3 kW. Also, find its value when the*

belt just begins to slip on the driving pulley, μ being 0.25 and the maximum tension in the belt 1200 N.

Solution

$N = 300$ rpm $a = 0.36$ m

$l = 0.8$ m $p = 3000$ W

(i) $P = (T_1 - T_2)v = \frac{Mgl}{2a} \times \omega r$

$$= \frac{Mgl}{2a} \times \frac{2\pi N}{60} \times r$$

$$3000 = \frac{M \times 9.81 \times 0.8}{2 \times 0.36} \times \frac{2\pi \times 300}{60} \times 0.18$$

$M = \underline{48.7 \text{ kg}}$

(ii) $T_1 = 1200$ N, $\mu = 0.25$, $\theta = \pi$ rad

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.25 \times \pi} = 2.19$$

$$T_2 = \frac{1200}{2.19} = 548\text{N}$$

$$T_1 - T_2 = \frac{Mgl}{2a}$$

$$1200 - 548 = \frac{M \times 9.81 \times 0.8}{2 \times 0.36}$$

$M = \underline{59.8 \text{ kg}}$

EPICYCLIC-TRAIN DYNAMOMETER

An epicyclic-train dynamometer is another transmission type of dynamometer. As shown in Fig. 15.21, it consists of a simple epicyclic train of gears. A spur gear A is the driving wheel which drives an annular driven wheel B through an intermediate pinion C. The intermediate gear C is mounted on a horizontal lever, the weight of which is balanced by a counterweight at the left end when the system is at rest. When the wheel A rotates counter-clockwise, the wheel B as well as the wheel C rotates clockwise. Two tangential forces, each equal to F, act at the ends of the pinion C, one due to the driving force by the wheel A and the other due to reactive force of the driven wheel B. Both forces are equal if friction is ignored. This tends to rotate the lever in the counter-clockwise direction and it no longer remains horizontal. To maintain it in the same position as earlier, a balancing weight W is provided at the right end of the lever. Two stops, one on each side of the lever arm, are used to limit the motion of the lever.

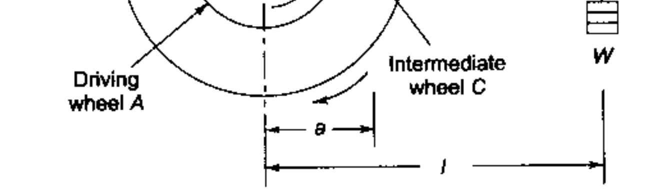


Fig. 15.21

For the equilibrium of the lever,

$$2\,F.a = W.l \quad \text{or} \quad F = \frac{Wl}{2a}$$

and torque transmitted $= F.r$ where r is the radius of the driving wheel

Thus power, $P = T.\omega = F.r.\frac{2\pi N}{60}$

15.12 BEVIS–GIBSON TORSION DYNAMOMETER

A Bevis–Gibson torsion dynamometer consists of two discs A and B, a lamp and a movable torque finder arranged as shown in Fig. 15.22(a). The two discs are similar and are fixed to the shaft at a fixed distance from each other. Thus, the two discs revolve with the shaft. The lamp is masked and fixed on the bearing of the shaft. The torque finder has an eyepiece capable of moving circumferentially. Each disc has a small radial slot near its periphery. Similar slots are also made at the same radius on the mask of the lamp and on the torque finder.

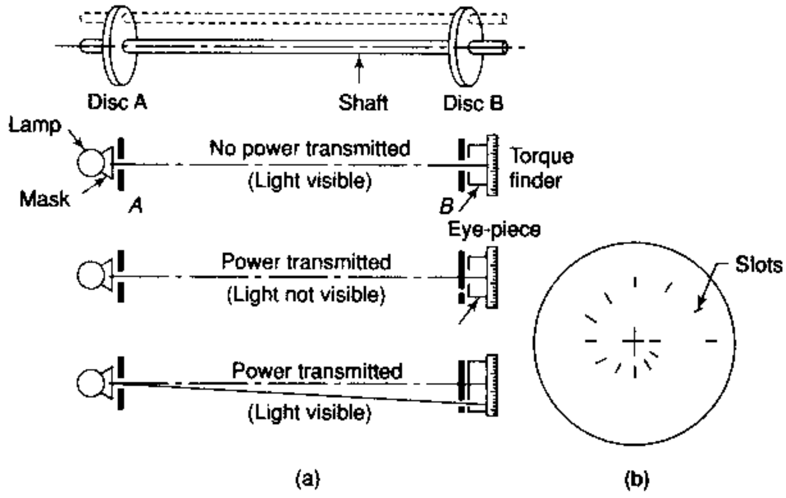


Fig. 15.22

When the shaft rotates freely and does not transmit any torque, all the four slots are in a line and a ray of light from the lamp can be seen through the eyepiece after every revolution. When a torque is transmitted, the shaft twists and the slot in the disc B shifts its position. The ray of light can no longer pass through the four slots. However, if the eyepiece is moved circumferentially by an amount equal to the displacement, the flash will again be visible once in each revolution of the shaft. The eyepiece is moved by a micrometer spindle. The angle of twist may be measured up to one hundredth of a degree.

In case the torque is varied during each revolution of the shaft as in reciprocating engines and it is required to measure the angle of twist at different angular positions, then each disc can be perforated with several slots arranged in the form of a spiral at varying radii [Fig. 15.22(b)]. The lamp and the torque finder have to be moved radially to and from the shaft so that they come opposite each pair of slots in the discs.

15.13 AUTOMOTIVE PROPULSION

The power required for propulsion of a wheeled vehicle depends mainly on the *tractive resistance*, i.e., the resistance faced by the vehicle on the road. The main components of the tractive resistance are the road resistance, aerodynamic resistance and gradient resistance.

1. Road Resistance

Road resistance consists of two types of resistances: rolling resistance and frictional resistance.

Rolling resistance Rolling resistance depends upon the condition of the road surface on which the vehicle is moving. For rail road its value is around 45–50 N per 1000 kg whereas for roads its value may vary from 80 to 250 N. However, for general purposes this can be assumed to be 150 N per 1000 kg. For cord tyres the value is approximately 2/3 of that for fabric tyres.

Frictional resistance Transmission losses like losses in the gear box, bearings, oil churning, etc., are included under frictional resistance. In direct gear, these losses are estimated at 10–12% and in a low gear at 15 to 20%. For private cars these figures may be taken somewhat lower.

2. Aerodynamic Resistance

Aerodynamic resistance is the resistance posed by air or wind and depends upon speed of the vehicle, its shape and the wind velocity. It can be taken as

$$R_a = kAV^2 \tag{15.21}$$

where R_a is the air resistance in N, k is a coefficient of air resistance, A is the projected area of the vehicle in m^2 and V is the vehicle speed in km/h. The usual value of k can be taken as 0.03 for average cars.

3. Gradient Resistance

It is dependent upon the weight of the vehicle and the gradient of the surface and is independent of the vehicle speed. Thus

$$R_g = Mg.\ G_r \tag{15.22}$$

where R_g is the gradient resistance in N, M is the mass of the vehicle in kg and G_r is the surface gradient and indicates the slope.

The sum of the road resistance, aerodynamic resistance and gradient resistance is known as *tractive resistance* (R_t)

Power required or demand power is

$$P = \frac{R_t.V}{\eta \times 1000}\ \text{kW} \tag{15.23}$$

where η is the transmission efficiency (from engine shaft to the wheel axle), R_t is the tractive resistance and V is the velocity in km/h.

Example 15.18 *A car with passengers has a mass of 1200 kg and a frontal area of 1.8 m^2. It is traveling up a gradient of 1 in 25 at a speed of 75 km/h. The rolling resistance of the car is given by R_r = (0.0112 + 0.00006 V) Mg and the air resistance coefficient is 0.02688. The engine develops 50 kW corresponding to an engine speed of 4500 rpm. The rear axle ratio is 5:1 and the transmission efficiency is 96%. If the wheel radius is 340 mm, determine the*

1. *tractive resistance*
2. *tractive effort available at the wheels and*
3. *acceleration while moving up the gradient*

Solution

M = 1200 kg A = 1.8 m^2

G_r = 1/25 = 0.04 P = 50 000 W

G = 5 η = 0.96

r_w = 0.34 m V = 75 km/hr

Tractive resistance,

$$\begin{aligned}R &= (0.0112 + 0.000\,06\,V)\,Mg + Mg.G_r + 0.026\,88\,AV^2\\ &= (0.0112 + 0.000\,06\,V + G_r)\,Mg + 0.026\,88\,AV^2\\ &= (0.0112 + 0.000\,06 \times 75 + 0.04) \times 1200 \times 9.81\\ &\qquad + 0.026\,88. \times 1.8 \times 75^2\\ &= 655.7 + 272.3\\ &= 928\ \text{N}\end{aligned}$$

$$\text{Brake power available} = T_e.\omega_e = \frac{T_w\omega_w}{\eta}$$

$$\begin{aligned}50000 &= (F_w.r_w).\frac{\omega_e}{G}.\frac{1}{\eta}\\ &= (F_w.r_w).\frac{2\pi N}{60G}.\frac{1}{\eta}\\ &= (F_w \times 0.34).\frac{2\pi \times 4500}{60 \times 5}.\frac{1}{0.96}\end{aligned}$$

F_w = 1498 N

Thus, force available for acceleration
$= 1498 - 928$
$= 570$ N

or $M \times f = 570$

or $1200 \times f = 570$

$f = 0.475$ m/s^2

$= \dfrac{0.475}{1000} \times 3600 = 1.71$ km/h/s

Example 15.19 *The resistance to motion is given by*

$$R_t = (0.011 + 0.000\,06\,V)\,Mg + 0.028\,AV^2$$

where M is the mass in kg, V is the velocity in km/h and A is the frontal area in m^2.

A jeep of 1400 kg mass and 2.4-m^2 frontal area is used to pull a trailor with a gross mass of 800 kg at 50 km/h in top gear on level road. If the jeep is capable of developing 40 kW of power for propulsion, find whether it is adequate for the job. The transmission efficiency may be taken as 92%. Also, find the pull on the coupling at this speed.

If all the power is used by the loading trailor, determine the pull in the coupling at 50 km/h and the load put on the trailor.

Solution

$M = 1400 + 800 = 2200$ kg $\qquad A = 2.4$ m^2

$P = 40\,000$ W $\qquad \eta = 0.92$

$V = 50$ km/h

$R_t = (0.011 + 0.000\,06\,V)\,Mg + 0.028\,AV^2$

$= (0.011 + 0.000\,06 \times 50) \times 2200 \times 9.81 + 0.028 \times 2.4 \times (50)^2$

$= 302 + 168$

$= 470$ N

$$\text{Brake power available} = \frac{F_w . v_w}{\eta}$$

$$40\,000 = \frac{F_w}{0.92}\left(\frac{50 \times 1000}{3600}\right)$$

$$F_w = 2650 \text{ N}$$

As F_w is quite large as compared to R_t, the jeep is adequate for the job.

Extra pull available $= 2650 - 470 = 2180$ N

The pull in coupling $= (0.011 + 0.00006\,V)\,Mg$ (assuming no wind resistance on the front of trailer)

$= (0.011 + 0.000\,06 \times 50) \times 800 \times 9.81$

$= 110$ N

Total pull at the coupling with extra load $= 2180 + 110 = 2290$ N

With extra load M

$2290 = (0.011 + 0.000\,06 \times 50) \times (800 + M) \times 9.81$

$800 + M = 16674$

$M = 15\,874$ kg

Example 15.20 *A truck is propelled in second gear up a gradient of 12%. The mass of the truck is 4400 kg, the speed is 32 km/h and the frontal area is 6 m^2. The tractive resistance of the truck is given by*

$$R_t = 0.015\,Mg + 0.038AV^2$$

where R_t is the tractive resistance in N, M is the mass in kg, A is the frontal area in m^2 and V is the velocity in km/h. Find the minimum power and the gear ratio in the second gear.

If the engine runs at 2400 rpm, what will be the minimum speed of this vehicle in the top gear on the level road if the efficiency is taken as 92% and the back axle ratio as 4.02? Also find the gear ratio in the top gear.

Solution

$M = 4400$ kg $\qquad A = 6$ m^2

$G_r = 0.12 \qquad \eta = 0.82$ and 0.92

$V = 32$ km/h $\qquad r_w = 0.4$ m

$= \dfrac{32 \times 1000}{3600}$

$= 8.889$ m/s

$$\omega_e = \frac{2\pi \times 2400}{60} = 80\pi$$

In the first case, gradient resistance is also to be considered.

$R_t = 0.015\,Mg + 0.038AV^2 + Mg.G_r$

$= (0.015 + G_r)\,Mg + 0.038\,AV^2$

$= (0.015 + 0.12) \times 4400 \times 9.81 + 0.038 \times 6 \times (32)^2$

$= 5827 + 234$

$= 6061$ N

Now,

$$\text{Brake power, } P = \frac{R_t.v}{\eta} = \frac{6061}{0.82}\left(\frac{32 \times 1000}{3600}\right)$$

$$= 67\,700 \text{ W or } 67.7 \text{ kW}$$

$$G = \frac{\omega_e}{\omega_w \times \text{Back axle ratio}}$$

$$= \frac{\omega_e}{(v/r) \times \text{Back axle ratio}}$$

$$= \frac{80\pi}{(8.889/0.4) \times 4.02}$$

$$= 2.81$$

In the top gear on a level road

$$R_t = 0.015\ Mg + 0.038AV^2$$

$$= 0.015 \times 4400 \times 9.81 + 0.038 \times 6 \times V^2$$

$$= 5827 + 234$$

$$= 6061 \text{ N}$$

$$P = \frac{R_t.v}{\eta}$$

$$67\,700 = \frac{(0.015 \times 4400 \times 9.81 + 0.038 \times 6 \times V^2)V}{0.92}$$

$$647.46V + 0.228V^2 = 224\,222$$

$$V = 90 \text{ km/h}$$

$$= \frac{90 \times 1000}{3600}$$

$$= 25 \text{ m/s}$$

$$G = \frac{\omega_e}{\omega_w \times \text{Back axle ratio}}$$

$$= \frac{\omega_e}{(V/r) \times \text{Back axle ratio}}$$

$$= \frac{80\pi \times 0.4}{25 \times 4.02} = 1$$

Summary

1. A brake is an appliance used to apply frictional resistance to a moving body to stop or retard it by absorbing its kinetic energy.
2. The functional difference between a clutch and a brake is that a clutch connects two moving members of a machine whereas a brake connects a moving member to a stationary member.
3. The main types of mechanical brakes are *block* or *shoe brake, band brake, band and block brake* and *internal expanding shoe brake*.
4. A *block* or *shoe brake* consists of a block or shoe which is pressed against a rotating drum. The force on the drum is increased by using a lever.
5. A *band brake* consists of a rope, belt or flexible steel band (lined with friction material) which is pressed against the external surface of a cylindrical drum when the brake is applied.
6. A *band and block brake* consists of a number of wooden blocks secured inside a flexible steel band which are pressed against the drum when the brake is applied.
7. An *internal expanding shoe brake* consists of two semi-circular shoes which are lined with a friction material such as *ferodo*. The shoes press against the inner flange of the drum when the brakes are applied.
8. The power required for propulsion of a wheeled vehicle depends mainly on the *tractive resistance*, i.e., the resistance faced by the vehicle on the road.
9. The main components of the tractive resistance are the *road resistance, aerodynamic resistance* and *gradient resistance*.
10. Road resistance consists of two types of resistances: *rolling resistance* and *frictional resistance*.
11. Aerodynamic resistance is the resistance posed by air or wind and depends upon the speed of the vehicle, its shape and the wind velocity.
12. Gradient resistance depends upon the weight of the vehicle and the gradient of the surface and is independent of the vehicle speed.

Exercises

1. What is a brake? What is the difference between a brake and a clutch?
2. What are various types of brakes? Describe briefly.
3. With the help of a neat sketch explain the working of a block or shoe brake.
4. What is meant by a self-locking and a self-energised brake.
5. Discuss the effectiveness of a band brake under various conditions.
6. Describe the working of a band and block brake

with the help of a neat sketch. Deduce the relation for ratio of tight and slack side tensions.

7. What is the advantage of a self-expanding shoe brake? Derive the relation for the friction torque for such a brake.
8. Discuss the effect of applying the brakes to a vehicle when
   (i) brakes are applied to the rear wheels only
   (ii) brakes are applied to the front wheels only
   (iii) brakes are applied to all the four wheels
9. What is meant by tractive resistance in case of wheeled vehicle? What are its main components?
10. Explain the following in case of a wheeled vehicle:
   (i) Road Resistance
   (ii) Aerodynamic resistance
   (iii) Gradient resistance
11. In a brake shoe applied to a drum Fig. 15.23, the radius of the drum is 80 mm and the coefficient of friction at the brake lining is 0.3. For the counter-clockwise rotation of the drum, determine the braking torque due to a force of 400 N applied at the end of the lever. (21.9 N.m)

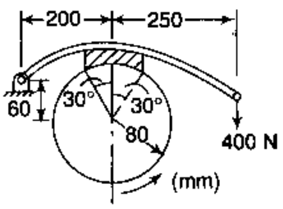


Fig. 15.23

12. Figure 15.24 shows a simple band brake which is applied to a shaft carrying a flywheel of 300-kg mass and of radius of gyration 280 mm. The drum diameter is 220 mm and the shaft speed 240 rpm. The coefficient of friction is 0.3. Find the brake torque when a force of 100 N is applied at the lever end. Also, determine the number of turns of the flywheel and time taken by it before coming to rest. (18.34 N.m, 64.5 rev, 16.13 s)

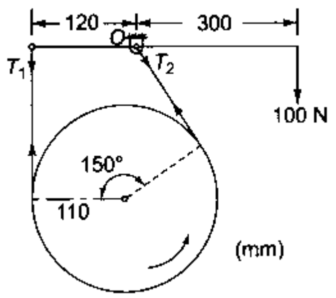


Fig. 15.24

13. For the shoe brake shown Fig. 15.25, the diameter of the brake drum is 400 mm and the angle of contact is 96°. The applied force is 3 kN on each arm and the coefficient of friction between the drum and the lining is 0.35. Determine the maximum torque transmitted by the brake. (1314 N.m)

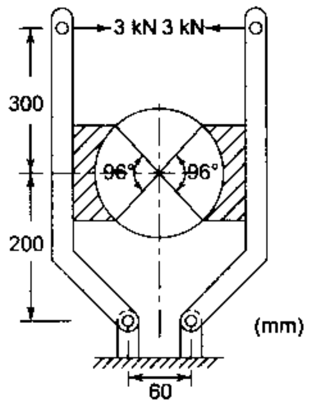


Fig. 15.25

14. A bicycle and rider having a mass of 120 kg and travel at 14 km/h on a level road. A brake is applied to the rear wheel of 900 mm diameter. The pressure on the brake is 110 N and the coefficient of friction is 0.05. What will be the distance covered by the bicycle and number of turns taken by its wheel before coming to rest? (164.9 m, 58.3)
15. Figure 15.26 shows the arrangement of a double block shoe brake. A turn buckle which has right

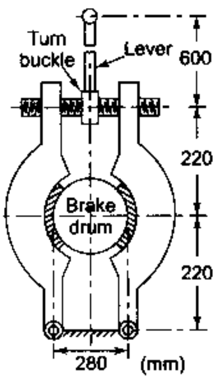


Fig. 15.26

and left-handed threads of six-start with a lead of 45 mm is used to apply the force to each block. The diameter of the turn buckle is 30 mm and it is rotated by a lever. Each block subtends an angle of 90° at the centre of the drum. The coefficient of friction for the brake blocks is 0.4 and for the screw and the nut is 0.15. Find the brake torque applied by a force of 120 N at the end of the lever.

(875.7 N.m)

16. The band brake shown in Fig. 15.27 is applied to a shaft carrying a flywheel of 300-kg mass with a radius of gyration of 400 mm and running at 340 rpm. Find the torque applied due to a pull of 100 N if $\mu = 0.25$. Also, find the number of revolutions of the flywheel before it comes to rest.

(794 N.m; 6.1 rev.)

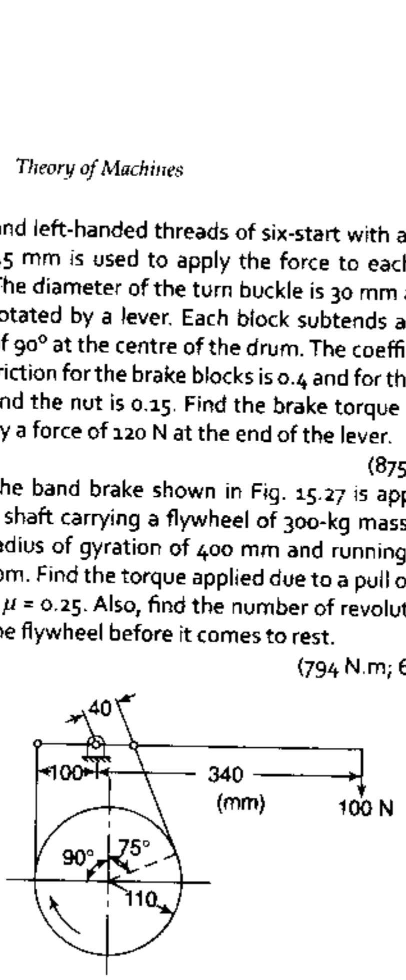


Fig. 15.27

17. A crane is used to support a load of 1.1 tonne on the rope round its barrel of 360 mm diameter (Fig. 15.14). The brake drum diameter is 560 mm, the angle of contact is 300° and the coefficient of friction between the band and the drum is 0.22. What will be the force F required at the end of the lever? Take $a = 150$ mm and $l = 800$ mm.

(1902 N)

18. A band and block brake has 10 blocks and each block subtends an angle of 15° at the centre of the wheel. The two ends of the band are fixed to pins on the opposite sides of the brake fulcrum at distances of 40 mm and 200 mm from it. Determine the maximum force required to be applied on the lever at a distance of 300 mm from the fulcrum to absorb 250 kW of power at 280 rpm. The effective diameter of the drum is 840 mm. Take $\mu = 0.35$.

(4440 N)

19. An internal expanding shoe brake has a diameter of 320 mm and a width of 30 mm. The cam forces are equal. Maximum pressure is not to exceed 80 kN/m². $\varphi_1 = 15°$, $\varphi_2 = 145°$, $a = 220$ mm, $c = 125$ mm and $\mu = 0.32$ (Fig. 15.16). Determine the actuating force and the brake torque.

(175.7 N; 48 N.m)

20. The following data refer to a car in which brakes are applied to the front wheels:
Wheel base = 2.8 m
Centre of mass from rear axle = 1.3 m
Centre of mass above ground level = 0.96 m
Coefficient of friction between road and tyres = 0.4
If the speed of the car be 40 km per hour, find the distance travelled by the car before coming to rest when the car
(i) moves up an incline 1 in 16
(ii) moves down an incline 1 in 16
(iii) moves on a level track

(22.5 m; 40.89 m; 29.03 m)

21. The following data refer to a laboratory experiment with rope brake:
Diameter of the flywheel = 1 m
Diameter of the rope = 10 mm
Dead weight on the brake = 50 kg
Speed of the engine = 180 rpm
Spring balance reading = 120 N
Find the power of the engine.

(3527 W)

22. In a belt transmission dynamometer (Fig. 15.20), the diameters of the driving and driven pulleys are 0.36 m and 0.8 m respectively. The power transmitted from the driving to the driven shaft is 20 kW. The speed of the driving shaft is 500 rpm. If $l = 1.2$ m and $a = 400$ mm, determine the weight on the lever.

(144.2 kg)